NECT

GRADE 4 - 9

SUBJECT MATHEMATICS

TERM 1 & 2 2019

TRAINER'S GUIDE

CONTENTS

Workshop Objectives

By the end of this training session, participants will:

1. Be aware of the programme for this training session.
2. Be informed of the Mathematics Teaching and Learning Framework (MTLF 2018).
3. Apply the requirements for a learning centred classroom.
4. Explain the process of conceptual understanding through content-based examples.
5. Understand the cognitive and social dynamics of learners age 9-15.
6. Be able to teach concept building in a learning centred classroom.

Before the Training

1. Be prepared to model excellence in training and facilitation.
2. Prepare the venue as best as possible, to ensure that participants are comfortable, that they can all see the presenter, and that the setup is conducive for discussion.
3. Be prepared to show the slide show and videos. Deal with technical issues before the training.
4. Be fully prepared, have all your materials laid out in an orderly fashion.
5. Display the objectives of the workshop and go through these with participants.
6. Display an 'agenda' – a chart listing every activity that will be completed, together with the planned time allocation.
7. At the end of every training day, reflect on the objectives and agenda, and tick off what has been achieved that day.
8. DISPLAY ALL RELEVANT RESOURCE THAT HAVE BEEN PRODUCED BY THE NECT FOR CLASSROOMS, I.E.: POSTERS; RESOURCE PACK ITEMS; ETC. (Make an effort to properly prepare these items to present them in a way that models good practice and will protect resources from wear and tear.)

Tone of the Training

1. Remember that you are training TRAINERS and TEACHERS. Please ensure that you address participants correctly.
2. Be polite, patient and RESPECTFUL always. This is possibly the most important aspect of being a trainer.
    - Participants will generally be open to you and to the programme if they are treated with respect.
    - Arrive early and be prepared – for every session!
    - Greet participants by name whenever possible and ensure that names are pronounced correctly.
    - Do not be dismissive of a participant's concern. If you do not have time, or if you know that the issue will be addressed later in the session, create a PARKING LOT. Write down the query and stick it in the parking lot to be addressed later.
    - Do not be dismissive of participants' knowledge, skills and experience. As much as possible, allow participants to contribute to discussions.
3. Remember that humour is always a good strategy – try to add some fun to the training, in a way that does not make anyone uncomfortable.
4. Please remember to use icebreakers and energisers when required – it is important to keep the mood and energy of the training positive.

NECT

GRADES 4 - 9 MATHEMATICS

TERM 1 & 2 2019 TRAINING PROGRAMME

	TIME	ACTIVITY	TEACHER WORKSHOP
1	30 minutes	Welcome, housekeeping and updates Introductions, reflections and agenda	
2	30 minutes	Pre-training activity	
3	30 minutes	Best Practice Mathematics Pedagogy	
4	1 hour 30 minutes	Conceptual understanding in a learning centred classroom – the basics and principles	
5	3 hours	Whole numbers (Grade 4 – 9)	
6	2 hours	Constructions (Grade 7 – 9)	
7	2 hours	2-D shapes and symmetry (Grade 4-6)	
8a	30 minutes	Selection of topics and preparation for participant presentations	
8b	3 hours	Presentations on conceptual understanding in a learning centred classroom	
9	1 hour 30 minutes	Orientation to trainer's guide	
10	1 hour	Training of teachers: planning session	
11	30 minutes	Post test	
10	30 minutes	Final questions and answers Closure and evaluation	

What you will need for this training:

ITEM	QUANTITY	CHECK
Flipchart stand and paper	1	
Kokis	10	
Blank A4 paper	100	
Laptop, data-projector and speakers	1	
USB with all materials	1	
Attendance register	1	
Prestik	5	
Evaluation Forms	1 per participant	
White board markers	1 per 2 participants	
Coloured A4 boards	3 per participant	
Pair of paper scissors	1 per participant	
15 cm ruler	1 per participant	
Exercise book (quad)	1 per participant	
Wax paper or plain paper	5-10 sheets per participant	

1	30 minutes	WELCOME, HOUSEKEEPING AND UPDATES INTRODUCTIONS, REFLECTIONS AND AGENDA	Facilitator:	What you will need: • Ensure that there is a sign outside your training room

1. Settle the group in plenary.
2. Welcome participants and complete the introductions.
3. Start the day with a short message or prayer if appropriate.
4. Share the relevant housekeeping notes, to ensure that participants are clear about the toilet and catering arrangements.
5. Present any relevant updates or share interesting and successful data or stories.

INTRODUCTIONS, REFLECTIONS AND AGENDA

1. Settle participants so that you have their attention.
2. If there are any new members of the group, or if you are new to the group, briefly do a round of introductions.
3. Next, tell participants that you would like to take some time to get them to reflect on their own experience of the implementation of the training and programme.
4. Make sure each participant has a piece of A4 paper.
5. Ask participants to fold the paper into 4.
6. Next, ask them to do the following:
   a. **In the first square**, **they must write:** their name, position, school or district.
   b. **In the second square**, **they must write:** one thing about the programme that is being successfully implemented in schools. Ask them to please write some details about this, even a short narrative to explain what is happening.
   c. **In the third square**, **they must write:** Something that is still problematic, that the programme has not managed to address. Ask them to write some detail about this, even a short narrative to explain what is happening.
   d. **In the fourth square, they must write:** Anything further that they still want from the NECT. Please point out that this cannot be resources.

6. Draw this diagram on flipchart paper to help participants remember what to do:

Name Position School or District	One thing that is working well in schools:
One thing that is still a problem in schools:	**One thing I think the NECT should do for my subject:**

2	30 minutes	PRE-TRAINING ACTIVITY	Facilitator: MQA	What you will need: • Copies of pre-test

1. Work together to hand out copies of the pre-training activity to participants.
2. Ask participants to not look at the activity yet.
3. Briefly explain the purpose of the pre-training activity, which is to measure the success of the training, not to measure the scores of individuals.
4. Briefly explain the text conditions, i.e.: to work independently and in silence, for a period of 30 minutes. Ask participants who finish before time to please cover their work and wait quietly for others.
5. As participants complete the pre-training activity, walk around and offer practical assistance if needed.
6. Once time is up, help to collect and collate pre-training activities in an orderly fashion.

3	30 minutes	BEST PRACTICE MATHEMATICS PEDAGOGY	Facilitator:	What you will need: • Flipchart papers • Marker pen

1. Settle participants so that you have their attention.
2. Explain to participants that you would like to start introducing them to the aspects of the best practice mathematics pedagogy as was requested by Subject Advisors and teachers for a long time now.
3. The purpose for this is that in this training we shall foreground certain best practice aspects of Mathematics teaching, that can gradually be considered and introduced.
4. Explain that you will give a presentation that introduces some of the crucial pedagogical principles of teaching mathematical concepts and how teachers can reach a learning centered classroom.
5. The presentation will include:
   a. Teaching mathematics for conceptual understanding.
   b. Promoting and nurturing a learning-centred classroom.
6. After the presentation, ask participants to share:
   a. Any questions of clarity.
   b. Any positive comments.
   c. Any ideas or recommendations.

4	1 hour 30 minutes	CONCEPTUAL UNDERSTANDING IN A LEARNING CENTRED CLASSROOM – THE BASICS AND PRINCIPLES	Facilitator:	What you will need: Quad paper exercise book

INTRODUCTION

1. No term captures completely all the aspects of knowledge, expertise, skills, understanding and competence in mathematics. We adopt (along with the modern trend in mathematics education globally) the term *mathematical proficiency* when we refer to what is necessary to master mathematics. Our understanding of mathematical proficiency (adapted from the original concept of the five strands of proficiency as coined by Kilpatrick, Swafford and Findell in their work *Adding it Up,* 2001) includes:
   a. Conceptual understanding – comprehension of mathematical concepts, operations and relations;
   b. Procedural fluency – the skill to carry out procedures flexibly, accurately, efficiently and appropriately;
   c. Strategic competence – the ability to formulate, represent and solve mathematical problems;
   d. Mathematical reasoning – the capacity for logical thinking, reflection, explanation and justification.
2. Up to now, much attention was spent on procedural fluency. The focus of our work in Term 1 & 2 this year is on conceptual understanding and the learning centred classroom – two concepts that need to be blended, as it is suggested that conceptual understanding is created in a learning centred classroom. The learning centered classroom is a mathematics learning environment where teachers and learners are both set to focus on the conceptual understanding of mathematics, and not solely on the teacher or the learner.
3. Ask participants to turn to page 4 in the training handout. As each of the following concepts are discussed, participants should make notes in their handout. Space is provided for writing one's own understanding of a concept before it is discussed as a group. More notes can then be added as required.

CONCEPTUAL UNDERSTANDING

A **concept** is the idea about something that exists, whether concrete or abstract. Similarly, a mathematical concept is a concrete or abstract idea of something that exists.

Concept formation of something like the properties of a 2-D kite shape is not difficult. It is often taught through cutting and handling a concrete 3-D paper kite as an object (discuss this). Concept formation of two dimensions is more complex though – that the kite shape on paper does not even include the paper as an object – that shape exists on a plane. Furthermore, to conceptualise for example, the two dimensions of a circle is completely abstract, because one can hardly form an idea of or conceptualize its length and width (breadth).

Conceptual understanding is knowing more than isolated facts and methods.

For example, the conceptual understanding of whole numbers includes more than knowing the bonds of ten, the multiplication facts (times tables), the various strategies for addition, subtraction, multiplication and division, being able to round correctly, etc. Conceptual understanding is to have an integrated/connected idea of the nature, magnitude and composition of number, the effect that the operations will have on the number, and how to explain that effect. One of the most tell-tale signs of conceptual understanding of whole numbers is the ability to estimate. Estimation is based on the idea in the learner's mind of the magnitude and change in numbers as a result of operating on them. Estimation is understanding at work. If estimation is properly done, it may include mental reasoning, but no written calculations. The proficient learner will be able to transfer conceptual understanding into new situations, apply it to new contexts and solve problems for which there is no set strategy.

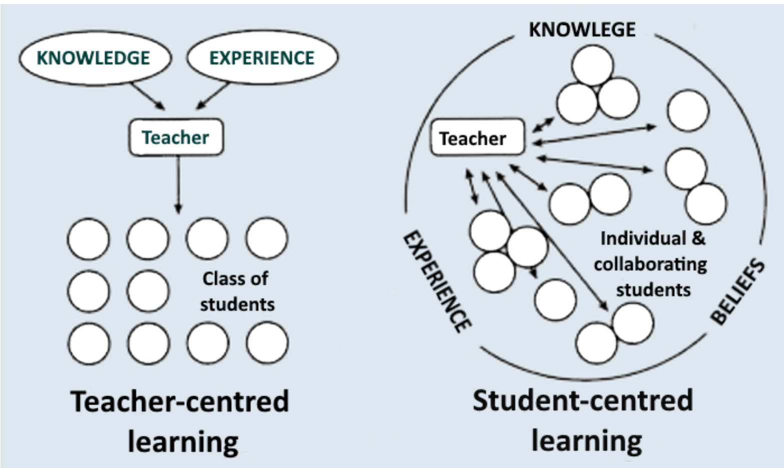
Facilitating concept formation is a teaching task for which there is not always direct guidance in the curriculum document. The required outcomes of the teaching and learning are set out systematically and learners are expected to use their understanding to solve problems, but exactly *how* conceptual understanding comes about and is cultivated, remains unclear. In this training we shall focus on the learning centred classroom as the key to concept formation.

DIFFERENT ORIENTATIONS TOWARDS TEACHING

Three main orientations towards teaching include a) the teacher centred classroom, b) the learner centred classroom and c) the learning centred classroom.

a) In the **teacher centred classroom** the teacher is the primary owner of knowledge and the main source of energy. He/she transfers knowledge as the more active partner and learners receive the knowledge as the more passive partners. The typical sequence in such a class is that the teacher will teach the concept (for example the properties of quadrilaterals), write on the board and/or hand out notes then give exercises to the learners to complete on their own. The teacher may even at this stage work at their desk while learners work quietly. It is not difficult to retain discipline in such a classroom because there is absolute structure. The teacher is in full control and learners are not allowed to voice opinions unless requested. The teacher can continue with her own work such as marking and administration while learners are doing the exercises. The shape best suited to depict this type of classroom is a pyramid, where the teacher is at the top, acting, and learners are at the bottom, reacting to the teachings and instructions. It is a top-down approach and politically resembles an autocracy where decisions and initiatives are mainly taken by the one at the top.

b) In the **learner centred classroom** learners are the primary owners of knowledge and the main source of energy. They create knowledge by investigation and exploration. The teacher acknowledges contributions as valid and should maintain structure at the lowest possible level to allow learners to explore freely. It is hard to retain discipline in such a classroom because there is virtually no structure, the teacher is not in control and learners may voice their opinions at any time. In the extreme practice of learner centred education no method or strategy can be enforced and there is no curriculum but what learners construct as knowledge. Learners' rights reign supreme and all their choices and decisions must be accommodated. The typical sequence in such a classroom is that learners will explore, come up with various findings and all get acknowledged. Class discussions may follow where some may be persuaded to other insights. No methods or strategies are prescribed, although the teacher may suggest some, but learners have the freedom of choice whether to accept them or not.



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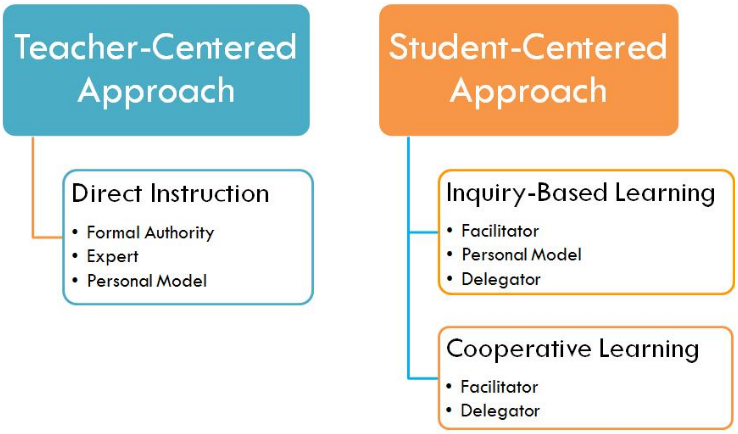

https://www.plaz-tech.com/technology-in-the-classroom-making-the-shift-from-teacher-centered-to-student-centered-approach/

c) In the **learning centred classroom** there is a distinct shift of focus away from people in the centre of the approach (subjective) to the mathematics concept as the core of all actions and activities (objective). All role players work towards the learning, understanding and mastering of a mathematical idea, no matter whether today the learner plays the larger part or tomorrow the teacher takes the larger part, as long as learning takes place and mathematical proficiency is reached.

The learning centred classroom has a few characteristics and requirements:

- It is **specific**: the focus must be precise, and it needs to be absolutely clear what the mathematical concept is that has to be developed.
- It is **intentional**: the purpose of the day must be clear to the teacher and to the learners and learners need to know what the outcomes are towards which they are working.
- It is **semi-structured**: because it is intentional and specific, the situation is structured and planned but the activities have the minimum structure to invite learners' innovative thinking.
- It is **meaningful**: there are no random activities that distract from the formation of the specific concept that is intended to be developed.
- It is **reciprocal**: there are different lines of connection – individual learners interact with the mathematical object, teacher and learner(s) interact, learners interact amongst themselves.
- It is **realistic**: the activities planned out for the formation of the concept originate, as far as possible, from the real world known to learners.
- It is **transcended**: once the concept has been formed, learners are practising the transcendence of the idea to other, related situations, both set up for them and by themselves.
- It is **assessed**: the process of concept development is continuously assessed to keep track of the level of understanding – informally by the teacher, self-assessment and peer assessment.

- It is **reasonable**: during the process and for all actions and decisions learners need to be able to explain *why.* In other words, be able to give reasons for doing what they do to develop logic and step-by-step thinking.
- It is **solidified:** once the concept is formed and understood it can be said it is still fluid. Now it needs to be practised over and over to ensure procedural fluency and ease of application.

NOTE

i. Concept formation can go wider and deeper than the curriculum and is not bound by absolute curriculum stipulations, for example "*we can only use three-digit numbers for addition, because this is what the curriculum prescribes*". Teachers must be able to discern the deep concept underpinning the topic in the curriculum. When the concept has been established, and there is clear understanding, the applications can be done within the confines of the curriculum.

ii. The same basic concept formation exercise can be done across a phase and is not necessarily bound to a specific grade, because the concept is underpinning both simple and complex applications – the concept does not change – its applications in different situations/contexts change, the exercises based on the concept are more complex and the level of abstraction is continuously raised in higher grades.

iii. The creation of the learning space must be developmentally appropriate, firstly as far as the cognitive development of learners are concerned, and secondly as far as the social dynamics of the specific age group is concerned, especially considering their gender preferences. The creation of a totally new concept can be supported with physical manipulatives, however learners in the Intermediate Phase are already in the semi-concrete phase of cognitive development, and understand representations of the concrete, as well as symbolic representations like number symbols. Learners in the Senior Phase can most certainly handle the symbolic representation of reality and they are moving towards the semi-abstract phase of cognitive development.

iv. It stays a point of discussion whether to group learners in same-ability groupings or in mixed ability groupings, since both have their advantages and disadvantages. The main concern in mixed ability grouping is that the stronger learner will dominate the weaker learner and the weaker learner, though slower, never reaches their own authentic understanding of the concept.

v. As will become clear from the example below, the formation of a concept is not something that is far away or only remotely related to mathematics. It uses mathematical terminology and processes, and when it is done there is not a huge distance to overcome to relate it to the exercises following.

vi. Clear and simple, step by step instruction is an important part of concept development; This allows the learners to focus on the concept and not to battle confusion and uncertainty about what is expected of them.

EXAMPLE OF CONCEPT FORMATION IN A LEARNING CENTRED CLASSROOM

Content Area: Patterns, Functions and Algebra

Topic: Number patterns

Phase: Intermediate Phase

Main concept: A number pattern is formed when numbers or numbers of objects follow a certain rule from one number to the next.

Learning space: Learners have a collection of countable objects in picture form amongst which they discover number patterns and establish the rules for each pattern. After they have done what they were given, they create their own pattern and peer-assess each other's pattern.

Information: The *number* of objects matter, not how they are configured – it is about *how many* of each object there are, not in which way or form they are arranged. The 🚲 pattern is descending.

Task 1: Inspect the objects in the table below and find the first four terms of six number patterns.

Write the patterns down in the correct form (e.g. 4, 7, 10, 13).

Extend the pattern by two terms (e.g. 4, 7, 10, 13, 16, 19).

Explain the rule of the pattern (*e.g. Starting at 4, the pattern increases by three each time*).

❋❋❋❋ ❋❋❋❋ ❋	🚲🚲🚲 🚲🚲🚲 🚲🚲🚲 🚲🚲🚲 🚲🚲🚲	●●●● ●●●● ●●	֍֍֍֍ ֍	☊☊☊☊☊ ☊☊☊☊☊ ☊☊☊☊☊ ☊☊☊☊☊ ☊☊☊☊	⌘⌘⌘⌘⌘ ⌘⌘⌘⌘⌘
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a) ❋:
b) 🚲:
c) ֍:
d) ●:
e) ☊:
f) ⌘:

Teacher's role: The teacher explains that a rule leads to a pattern, and what ascending and descending patterns are. She does rounds, informally assesses and assists where needed.

Assessment: The formative assessment for this exercise is firstly informal observation and intervention by the teacher while the first task is in progress. The focus points to look out for are -

- ✓ Do learners master writing down the sequences in order?
- ✓ Do learners explain the rules saying where the pattern starts and how it progresses?
- ✓ Do learners manage to extend the patterns?

The assessment for learning in Task 1 is informal by the teacher on her rounds as explained and then a simple self-assessment by learners with the correct answers provided by the teacher as follows:

a)	🕷 :	3, 5, 7, 9, **11, 13…** Starting at 3, the pattern increases by 2 each time
b)	🚲 :	15, 12, 9, 6, **3, 0…** Starting at 15, the pattern decreases by 3 each time
c)	֍:	5, 9, 13, 17, **21, 25…** Starting at 5, the pattern increases by 4 each time
d)	●:	$2\frac{1}{2}$, 5, $7\frac{1}{2}$, 10, $\mathbf{12\frac{1}{2}}$, **15** … Starting at $2\frac{1}{2}$, the pattern increases by $2\frac{1}{2}$ each time
e)	☊:	3, 6, 12, 24, **48, 96…** Starting at 3, the pattern is doubled each time / is multiplied by 2 each time
f)	⌘:	1, 3, 6, 10, **15, 21…** Starting at 1, the pattern increases by 2, then 3, then 4 and so on

Note: The proof of concept formation and conceptual understanding is when learners can create, describe and interpret their own patterns according to a rule.

Task 2: Learners (individually) develop the first four terms of their own patterns in the space below:

- Pattern 1 uses multiplication in its rule.
- Pattern 2 is a descending pattern.
- Pattern 3 uses addition of whole numbers in its rule.
- Pattern 4 uses subtraction of fractions in its rule.

Learners (in pairs) hand their patterns to their partners to describe it in symbols and words.

Learners peer-assess each other's patterns, their descriptions and the formulation of the rules.

Pattern 1: Pattern with multiplication in its rule:					
Number pattern: **Rule of the pattern:**					

Pattern 2: Descending pattern:					
Number pattern: **Rule of the pattern:**					

Pattern 3: Pattern with addition of whole numbers in its rule:					
Number pattern: **Rule of the pattern:**					

Pattern 4: Pattern with subtraction of fractions in its rule:					
Number pattern: **Rule of the pattern:**					

5	3 hours	WHOLE NUMBERS (Grade 4-9)	Facilitator:	What you will need:

INTRODUCTION

- We are now going to apply the principles of conceptual understanding in a learning centred classroom to a cluster of topics that will be taught in Term 1 and 2 across the Intermediate- and Senior Phases.
- The focus will be on four fundamental concepts in the overarching topic: Whole Numbers, namely
- Number sense: *How numbers follow on each other*.
- Number sentences: *Writing numbers and operations in ordered ways to reach a certain result.*
- Addition and subtraction of whole numbers: *Estimation of the magnitude of change when a number is enlarged through adding or doubling and when it is reduced through subtracting or halving.*
- Multiplication and division of whole numbers: *Multiplication and division are inversely related and interchangeable.*
- Below are suggestions for the creation of a learning centred classroom to establish and promote the understanding of the above concepts.

Number sense: *How numbers follow on each other*

The main concept that learners must understand in this topic, is how numbers follow on from each other. Learners in the 9-12 years old age range are in the semi-concrete phase of cognitive development and work well with diagrams and symbols like numbers. Learners over 12 years have progressed to the symbolic- to semi-abstract phase of cognitive development. Our pitch is in the centre, namely on the representation of number as symbols. In the Foundation Phase the focus was on counting, how objects correspond with numbers and how to allocate symbols to several objects. At this stage, learners must develop a working model for themselves to reflect smaller and larger numbers from left to right and from bottom to top. Conceptualising whole numbers from left to right and from bottom to top prepares learners for the concept of the Cartesian plane.

There are two sets of instructions that apply for Intermediate Phase and Senior Phase respectively. The first set is simple and can also be given remedially for struggling learners in the SP. The second set is more complex and can also be given as enrichment for fast IP learners. Two fraction questions are included in each set.

Provide each learner with six different open number lines, two continuous, two dotted and two dashed. Ask them to cut the page in half and use three lines vertically and three horizontally to write their numbers:

Simple instructions: On their number lines, learners use one line each time to show the following:

- Counting in twos
- Marking out odd and even numbers
- Counting backwards from twenty in threes
- Counting in fives from two
- Indicating $14\frac{1}{2}$ on the second (dotted) line
- Showing the quarters of twenty

Complex instructions: On their number lines, learners use one line each time to show the following:

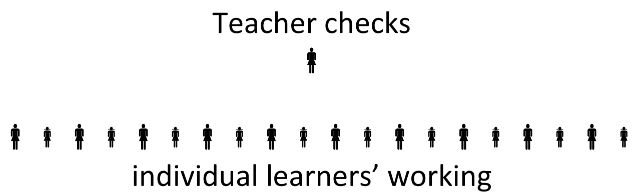
- Counting in multiples of sevens between 60 and 80
- Marking the prime numbers between 35 and 57
- Showing the factors of 24
- Showing the first three powers of 3
- Indicating $\frac{4}{5}$, $\frac{8}{5}$, $\frac{12}{5}$, $\frac{16}{5}$ on the second (dotted) line
- Dividing 35 by 6

The teacher needs to make sure that each learner's number lines are workable (functional) and that they can do all six activities above using their two sets of number lines.

Assessment for learning

The focus in the best practice for mathematics teaching and learning is on assessment FOR learning, rather than assessment OF learning. This means that teachers are already assessing while learning is underway, and not at the end of the learning only.

In this topic, the teacher will already see where understanding is and where misconceptions are while learners are constructing and using their number lines. This gives her an opportunity to direct and guide the learning – which means she has assessed FOR learning while the learning was underway. Later, after learning has taken place, that is in the test or examination, she will be doing assessment OF learning.



Number sentences: ***To write numbers and operations in ordered ways to reach a certain result.***

The main concept in this part is *that you can write down numbers and operations in various ordered ways to reach a certain result*.

Teachers create a learning space where a certain group of numbers and the addition and subtraction operation signs are available for learners to configure in their own way to reach a given result. During the developmental stage above 9 years old, learners can work with numbers out of any context (context free). Various solutions of learners may differ, and each response should be considered on its own merit. Learners are given the sheet of paper below in pairs. They take turns in completing and checking number sentences: When A completes, B checks; when B completes, A checks.

You have only two operation signs (+, –) and the following numbers to work with. In some items the operations are given; in some the answers only are given:

23	15	7	13	6	12	37	5	25	18	17	23
4	16	22	11	1	34	25	4	19	10	0	39

A		+		=	22		11				
B		+	13	=	12						
A		–		=	1						
B		–		=	1						
A				=	30						
B				=	30						
A				=	0						
B				=	0						
A		+		=	1		19		4		
B		+		=	1		18		6		
A		–		=	1						
B		–		=	1						
A	(	+	)	+	16		(16		)	+	
B	7	+	(	+	)		18		(	+	)
A		–		=							
B		–		=							
A		+		=							
B		+		=							
A		–		=							
B		–		=							

The teacher needs to give individual guidance where learners struggle with the number sentence concept. The last four items are especially challenging and can well be done in Senior Phase only. Key to any explanation is the equal sign and the balance of the left-hand side of the equation with the right-hand side.

Assessment for learning

In this exercise it is important to assess whether the following learning is taking place:

- Is there a clear understanding of the concept of equations – that the left-hand side and the right-hand side of the number sentence must have the same value?
- Is there and understanding of the laws of number, specifically the commutative and associative properties of addition, and how they do not apply to subtraction?
- Is the concept of **consecutive numbers** clear – that there is a difference of one between any two consecutive numbers?
- Is the **property of zero** clear?
- Is the **order of subtraction** clear?

We make use of peer assessment, with teacher guidance only where needed. Peers must explain to the second partner why they say something is wrong or right. The objective is that through the cross checking of peers, learners will start to develop the idea of self-assessment.

Learners check each other's work in pairs

Teacher supports

Addition and subtraction: estimating the magnitude of change when a number is enlarged through adding or doubling and when it is reduced through subtracting or halving

For the development of this concept, teachers create a learning space where learners use a model to develop their own sense of estimating in addition or doubling and in subtraction or halving. It is not necessary to be precise – in fact it is not about being precise through calculating, but about developing a sense of the enlargement or reduction without calculating. Calculation is used to check the reasonability of the estimation.

The model that we use, is a number line, which is a spatial model – it has length – larger numbers take up more space and smaller numbers less. Learners are given the strips below in groups of four. They follow the instructions for each of eight items and then compare and discuss their group's results. Each group then presents their best estimation to the whole class.

Use the eight strips below, each with instruction and number line. Follow the instructions.

The rule: **Nobody may calculate the answer**. The answers must be represented by a vertical line on the strip:

1. Estimate and draw a line where you think the answer will be to: 246 + 89

0	50	150	200	250	300	350	400	450	500	550	600

2. Estimate and draw a line where you think the answer will be to: 436 + 637

0	100	200	300	400	500	600	700	800	900	1000	1100

3. Estimate and draw a line where you think the answer will be to: 555 + 555

0	100	200	300	400	500	600	700	800	900	1000	1100

4. Estimate and draw a line where you think the answer will be to: 442 - 67

0	50	150	200	250	300	350	400	450	500	550	600

5. Estimate and draw a line where you think the half of 238.

0	20	40	60	80	100	120	140	180	200	220	240

6. Estimate and draw a line where you think the answer will be to: 38 + 83 - 42

0	10	20	30	40	50	60	70	80	100	110	120

7. Estimate and draw a line where you think the answer will be to: 53 – 29 + 88

0	10	20	30	40	50	60	70	80	100	110	120

8. Estimate and draw a line where you think the answer will be to: 155 + 236 + 341

0	100	200	300	400	500	600	700	800	900	1000	1100

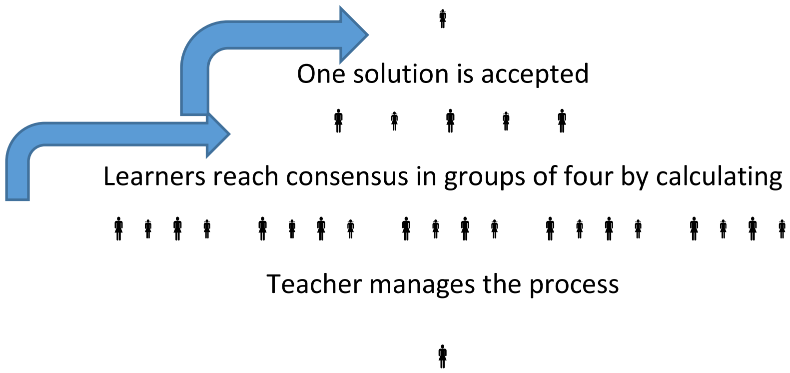
The teacher needs to keep an eye on the process until the groups have reached a consensus. When everybody has finished estimating, each group calculates the exact answer and learners compare their markings to the exact answers.

Assessment for learning

The formative assessment model for this exercise can be depicted as a pyramid: all learners do the instructions individually, the groups reach consensus through calculation and then the class accepts the best solutions, by which each learner judges their own answers to justify or rectify their own findings. We want to ensure that the sense of the magnitude of change is developed -

- When a smaller number is added to a larger number.
- When a larger number is added to a smaller number.
- When a number is enlarged by the same number.
- When a smaller number is subtracted from a larger number.
- When a number is split in two parts of equal size.
- When a number is added to a number and a number is subtracted from the sum.
- When a number is subtracted from a number and another is added to the difference.
- When two numbers are added to a number.

The assessment for learning in this exercise makes use of group – and individual assessment. The objective is that learners check their solutions by calculation and then reach consensus as a class.



The table above can be used by teachers to set up their own memorandum for this exercise – calculate the correct answer and draw the line where the exact answer should be.

Multiplication and Division: multiplication and division are inversely related and interchangeable.

For the development of this concept, teachers create a learning space where learners are given a situation which requires various operations. We promote heuristics in this exercise, meaning that we do not prescribe the order or methods of calculation, as long as a logical process leads to an acceptable solution. Our conceptual development space centres around a single situation, which we shall explore from various angles and using various models.

My name is __________. I once had a task in school. We were writing Departmental tests and I had to make up packets of 6 sheets of paper for each learner in the Phase. There were 13 examination pads in the store room. Each examination pad had 80 pages. There were 195 learners in the Phase. In the end I had to ask Teacher Ntombi to buy more examination pads because there were not enough sheets for 195 learners. Please help me to check how many more examination pads she had to buy.
.

The teacher needs to keep an eye on the process and allows learners their own methods and their own order of calculating how many more examination pads were needed. It does not matter whether they divide or multiply to find their solutions – in fact the interrelatedness and interchangeability of the two operations is the concept to be developed. In the end, the teacher consolidates by inviting learners to write their processes on the board and writing her own calculation on the board too. It does not matter how learners reached their answers, as long as they followed a logical process and calculated correctly.

The teacher talks through the systematic logical thinking process. Learners check their own thinking process and identify where they went right or wrong. Teacher may want to ask learners to come to the front to do the calculations on one side. Her task is to guide learners through the logical reasoning process, as follows:

Teacher discussion	**Learner calculations**
"There were 13 exam pads in the store room." "Each exam pad had 80 pages." **Grade 4's – how shall we find out how many pages we have all together?** "So there were *13 x 80 = 1040 pages* all together" **Grade 4's – how shall we find out how many packets can I make up from 1040 pages?** "I made up packets of 6 sheets, so I made up *1040 ÷ 6 = 173 packets, 2 sheets remaining.*" "There were 195 learners. There were only enough packets for 173 learners." **Grade 4's – how can we find out how many more packets I need?** "So I needed packets for *195 – 173 = 22* more learners." **Grade 4's – how shall we find out how many more pages do I need for 22 packets?** "Packets for 22 learners would be *22 x 6 = 132 sheets*". **Grade 4's – how shall we know how many more exam pads I need if I need 132 more sheets?** "If Teacher Ntombi bought 2 exam pads, it would be enough, because one would not be enough *2 x 80 = 160*".	

This exercise uses **collaborative formative assessment**. The teacher weaves the logical reasoning together, but the suggestions and calculations come from the class. Learners peer-check and self-check calculations done on the board.

GRADE 7 – 9

INTRODUCTION

1. The Intermediate phase covered content and suggestions for the creation of a learning centred classroom to teach the content conceptually.
2. For the Senior phase training, instead of the focus being on content as in previous training, we are going to look at some common misconceptions that learners have regarding whole numbers which will ultimately lead us to rational numbers as well.
3. Looking for patterns in numbers will also be covered.

Background

1. According to Daniel Willingham, successful mathematics learning requires three different abilities that must be developed and woven together. These are:
   - control of facts
   - control of processes
   - conceptual understanding
2. He also notes that instruction for understanding must be different than the learning of basic skills and facts. Learning a new concept is dependent on what the learner already knows. As a learner advances through the Senior phase and into the FET phase, new concepts will increasingly depend on conceptual knowledge.
3. For example, understanding algebraic equations depends on the right conceptual understanding of the equal sign. If learners fail to gain conceptual understanding, it will become more and more difficult to catch up, as new conceptual knowledge depends on the old. Learners will become more and more likely to simply memorize algorithms and apply them without understanding.
4. To achieve conceptual understanding requires carefully thought out strategies that will ask of the learner to bring their past experience into present work and then use these to learn a new concept by connecting their experience to understanding.
5. Today, we are going to look at a few common misconceptions or misunderstandings that the learner could be bringing to the classroom.
6. Ask participants to turn to page 13 in the handout where they will find space to note their own thoughts as each of the following misconceptions is discussed:

Multiplication always results in a bigger number and division always results in a smaller number

- State two cases where this is true
- State two cases where this is not true
- Why do you think learners think this?
- What could the teacher do to alleviate the problem? Discuss with the participant next to you.

Once participants have had a chance to complete their examples and answers, ask for everyone's attention. Spend a few minutes having a group discussion. The focus must be on the two questions. If there are not enough contributions or if the points below are not mentioned, use them to create further discussion.

Examples where this is true:

$5 \times 7 = 35$ $26 \div 2 = 13$

Examples where this is not true:

$\frac{1}{4} \times 12 = 3$ $6 \div \frac{1}{2} = 12$

Why do many learners believe that multiplication always results in a larger number?

- Our everyday language implies this. For example, the weeds in the garden multiplied within days.
- The word multiple is also used in the English language to imply many.
- Learners' first encounters with multiplication is with Natural numbers where this statement is true.
- Repeated addition is often used as an explanation of multiplication.

Why do many learners believe that division always results in a smaller number?

- This is how it always worked for them when they first encountered division.
- The word "share" is often used (not incorrectly) in earlier grades and when something is shared there is always a smaller amount to go around.
- Visualising a question like $\frac{3}{4} \div \frac{2}{5}$ is very difficult.

What could the teacher do?

For multiplication:

- Using the word 'of' could help (Instead of 2 x 3 say 2 of 3 which would be the same as two lots of three')
- Use patterns showing that although the rule is working in the beginning when natural numbers are used, once fractions are used it is no longer true. For example:

$$3 \times 4 = 12$$
$$2 \times 4 = 8$$
$$1 \times 4 = 4$$
$$\frac{1}{2} \times 4 = 2$$

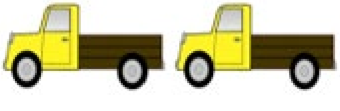
- Ask learners to investigate for themselves by asking them 'If you multiply two numbers, how can you tell whether the product is larger or smaller than the two factors?' Learners could work in pairs and the teacher could first pose the question and allow learners to have a discussion for a few minutes. Then learners could use their calculators to test several multiplication questions and test their initial 'theory'.

For division

- Be aware that the use of the word sharing could be confusing for learners.
- Use easy examples that don't require calculations to show why you could divide and get a larger number.

For example:

1. We can share equally and get bigger numbers without confusion if learners understand the concept. A half ton sand truck breaks down and the owner must send a replacement, but he has only quarter ton trucks. How many must he send so they can share the sand equally between the quarter ton trucks? Learners instinctively say two and only then one can show them how to divide fraction by fraction and how it is possible to derive a whole number.



2. How many halves do I have in 4 cakes $\left(4 \div \frac{1}{2}\right)$?

A number with three digits is always bigger than one with two

- State two cases where this is true
- State two cases where this is not true
- Why do you think learners think this?
- What could the teacher do to alleviate the problem? Discuss with the participant next to you.

Once participants have had a chance to complete their examples and answers, ask for everyone's attention. Spend a few minutes having a group discussion. The focus must be on the two questions. If there are not enough contributions or if the points below are not mentioned, use them to create further discussion.

Examples where this is true:

$456 > 97$ $\qquad$ $111 > 88$

Examples where this is not true:

$0{,}5 > 0{,}45$ $\qquad$ $1{,}9 > 1{,}888$

Why do learners believe that a number with three digits will always be larger than one with only two digits?

- This is how it always worked when they first encountered ordering of numbers.
- They do not have a good understanding of place value.

What could the teacher do?

- Spend more time on asking questions about place value in both whole numbers and decimal fractions.

To multiply by powers of 10, just add as many zeroes as there are 10's

- State two cases where this is true
- State two cases where this is not true

- Why do you think learners think this?
- What could the teacher do to alleviate the problem? Discuss with the participant next to you.

Once participants have had a chance to complete their examples and answers, ask for everyone's attention. Spend a few minutes having a group discussion. The focus must be on the two questions. If there are not enough contributions or if the points below are not mentioned, use them to create further discussion.

Examples where this is true:

$5 \times 10 = 50$ $\quad\quad$ $526 \times 10^3 = 526\ 000$

Examples where this is not true:

$\frac{1}{2} \times 10 = 5$ $\quad\quad$ $0{,}34 \times 10 = 3{,}4$

Why do learners believe that adding zeroes when multiplying by powers of 10 is the rule?

- This is how it always worked for them when they first encountered multiplying natural numbers by powers of 10.
- Someone has taught them this 'easy' shortcut.

What could the teacher do?

- Ensure learners have a sound understanding of place value.
- When dealing with decimal fractions don't talk about the decimal comma moving – it is a fixed point. (For younger grades, number sliders can be used. A resource is available in the handout)

Why can't we divide by zero?

Ask participants to work with a partner and answer this question. Tell them to find a way to explain this to learners at senior phase level.

Once participants have had a chance to complete what they think is a good way to answer the question ask for everyone's attention. Ask for 2 volunteers to show their colleagues what they think could be done. After both volunteers have demonstrated their method, ask the group for comment.

Here are two possible demonstrations:

1. <u>Word problems</u> - Find some simple word problems that demonstrate division.

Example 1

You have 4 cakes and want to share them among 2 people	Each person will get 2 cakes $(4 \div 2 = 2)$
You have 4 cakes and want to share them among 8 people	Each person will get $\frac{1}{2}$ of a cake $\left(4 \div 8 = \frac{1}{2}\right)$
You have 0 cakes and want to share them among 2 people	Each person will get no cake $(0 \div 2 = 0)$
You have 4 cakes and want to share them among 0 people	There is no-one to get cake. This is impossible. $(4 \div 0 = \text{undefined})$

Note that the key idea for this method is to use several word problems that are fairly easy to work out without needing a pencil and paper. The calculations were shown above but these are not the point of the exercise.

Example 2

You have a 2m long piece of sugar cane and want to cut it into equal pieces of 20cm in length. How many pieces will you get?	There will be 10 pieces of 20cm each $(200 \div 20 = 10)$
You have a 2m long piece of sugar cane and want to cut it into 5 equal pieces. How long will each piece be?	Each of the 5 pieces will be 40cm long $(200 \div 5 = 40)$
You have a 2m long piece of sugar cane and want to cut it into 0 equal pieces. How long will each piece be?	This is not possible. I cannot take a real item and reduce it to zero pieces. $2 \div 0 =$ ☹

Example 3 (this would work well in Grade 8 or 9)

The smaller a fraction is, the closer it moves to zero.

The smaller the fraction divisor is, the more parts we get when we divide a whole number. Look at this:

$2 \div \frac{1}{2} = 4$ $2 \div \frac{1}{3} = 6$ $2 \div \frac{1}{4} = 8$ $2 \div \frac{1}{5} = 10$ $2 \div \frac{1}{6} = 12$ $2 \div \frac{1}{8} = 16$ $2 \div \frac{1}{12} = 24$

$2 \div \frac{1}{20} = 40$ $2 \div \frac{1}{600} = 1\,200$ $2 \div \frac{1}{800000} = 1\,600\,000$ $2 \div \frac{1}{123456789} = 246\,913\,578$

$2 \div \frac{1}{987654321000000} = 1\,975\,308\,642\,000\,000$

$2 \div 0 =$ an infinitely large number which we cannot define. It is undefined.

The above example leads to an understanding of limits for functions and Calculus in the FET phase.

2. Inverse operations

Division is the inverse operation of multiplication. If we can multiply, then we can divide by reversing the numbers. If we can divide, then we can multiply by reversing the numbers.

$2 \times 8 = 16$ which means → $16 \div 2 = 8$ or $16 \div 8 = 2$

$10 \times 12 = 120$ which means → $120 \div 12 = 10$ or $120 \div 10 = 12$

$\frac{14}{7} = 2$ which means → $2 \times 7 = 14$

$\frac{0}{6} = 0$ which means → $0 \times 6 = 0$

Now move on to the issue at hand:

$\frac{5}{0} = ?$ which should mean → The answer multiplied by zero will equal 5. There is no number that can be multiplied by zero and give anything other than zero. This division is impossible.

$\frac{9}{0} = ?$ which should mean → The answer multiplied by zero will equal 9. There is no number that can be multiplied by zero and give anything other than zero. This division is impossible.

Why does 'tip and times' work when dividing fractions?

Ask participants to work with a partner and answer this question. Tell them to find a way to explain this to learners at Senior Phase level.

Once participants have had a chance to complete what they think is a good way to answer the question ask for everyone's attention. Ask for a volunteer to show their colleagues what they think could be done. Ask the group for comment.

Here is an explanation:

When multiplying fractions, we multiply numerators with numerators and denominators with denominators. When dividing, it would seem intuitive to do the same – divide numerators with numerators and denominators with denominators. And we can!

This demonstration shows us why 'invert and multiply' or 'tip and times' really works:

Example:

$$\frac{2}{5} \div \frac{3}{4}$$

$$\frac{2}{5} \div \frac{3}{4} = \frac{2 \div 3}{5 \div 4}$$

$$= \frac{\frac{2}{3}}{\frac{5}{4}}$$

This is the part where some knowledge of equivalent fractions is required. Fractions on top of fractions (compound fractions) can look messy. We need to sort that out. If we multiply the numerator and denominator by 3 (the denominator in the fraction at the top), the fraction will not change (since $\frac{3}{3} = 1$ and multiplying by 1 does not change the number being multiplied).

Let's try that:

$$= \frac{\frac{2}{3} \times 3}{\frac{5}{4} \times 3}$$

$$= \frac{2}{\frac{15}{4}}$$

We still have a compound fraction. The denominator of the bottom fraction is 4 – so now we will multiply both the numerator and denominator by 4.

$$= \frac{2 \times 4}{\frac{15}{4} \times 4}$$

$$= \frac{8}{15}$$

Let's check if we get the same answer with the shortcut we know:

$$\frac{2}{5} \div \frac{3}{4}$$

$= \frac{2}{5} \times \frac{4}{3}$

$= \frac{8}{15}$ ☺

This still doesn't explain why it works. Let's have a closer look at what was actually done in the longer version of the calculation:

$$\frac{2}{5} \div \frac{3}{4} = \frac{2 \div 3\ (\times 3 \times 4)}{5 \div 4\ (\times 3 \times 4)} = \frac{8}{15}$$

With a knowledge of inverse operations, we know that $\div\ 3$ and $\times\ 3$ are inverse operations so they 'cancel' each other out. If you divide by 3 then multiply by 3 you will be back where you started. $\div\ 4$ and $\times\ 4$ are also inverse operations.

Let's look at what happens when we take these out of the above equation:

$\frac{2}{5} \div \frac{3}{4} = \frac{2 \times 4}{5 \times 3} = \frac{2}{5} \times \frac{4}{3}$ which is what is known as the 'shortcut'!

7. Tell participants that we have come to the end of our look at common misconceptions and how we could try and avoid these by changing the way we teach a concept in the classroom.
8. It is often difficult to know whether a learner truly understands a certain concept. One might say that the only observable behaviour that will show us whether the learner understands a concept is whether the learner transfers that knowledge correctly or efficiently into new situations. If transfer is the measure, then conceptual understanding could get entangled with memory, because a learner's chances of correctly transferring a concept is partly determined by his/her ease of recalling it.

6	2 hours	CONSTRUCTIONS (Grade 7-9)	Facilitator:	What you will need: • Wax paper or other paper • Ruler • Marker

INTRODUCTION

1. We are now going to apply the principles of conceptual understanding in a learning centred classroom to Constructions in the Senior phase.
2. The focus will be on an overall look at constructions from a different perspective. The following will be covered:
    - Why are constructions necessary?
    - Teaching constructions through paper folding to lead to a better conceptual understanding
    - Included are suggestions for the creation of a learning centred classroom to establish and promote the understanding of the above concepts.

Definition of geometric constructions:

1. The drawing of various shapes using only a pair of compasses and straightedge or ruler. No measurement of lengths or angles is allowed. The word construction in geometry has a very specific meaning: the drawing of geometric items such as lines and circles using only compasses and straightedge or ruler.
2. Tell participants that even though the correct way to do geometric constructions is with a ruler and compass, today we are going to look at another way to teach constructions to give the learner an opportunity to gain conceptual understanding of geometric shapes which will lead to a better understanding of the theorems taught in the phase as well.

Discussion: Why do we need to teach constructions?

Ask participants for their input on the question. Over and above the input of the participants, ensure the following points are covered:

3. Euclid first stated many of his theorems in terms of constructions. His axioms (a statement or proposition which is regarded as being established, accepted, or self-evidently true) are closely related to the tools he used for construction. Just as his axioms and postulates (to suggest or assume the existence, fact, or truth of something as a basis for reasoning, discussion, or belief) let us prove

everything with a minimum of assumptions, a compass and straightedge let us construct everything precisely with a minimum of tools.

4. Learning to construct geometric concepts such as angles, quadrilaterals and perpendicular lines assists learners in developing an understanding of the theorems to be learnt and the properties of quadrilaterals. For example, the skills you need to construct a square without a protractor, are closely related to the thinking skills you need to prove theorems about squares.
5. Geometry is a skill required to enable learners to continue with Core Mathematics to Grade 12. Without conceptual understanding in the senior phase, passing Grade 12 mathematics will be unlikely. It counts 27% of the final Paper 2 and is also used in Analytical Geometry and Trigonometry so in reality it is worth even more.
6. Even if learners do not continue with Core Mathematics, learning geometry still teaches something of importance to all of us: When we know how to apply and understand the relationship between shapes and sizes, we will be better prepared to use them in our everyday lives. Geometry will assist us in doing that, because it provides the knowledge of how to deal with measurements and relationships of lines, angles, surfaces and solids**.**

Discuss with participants what a lesson on constructions should be like:

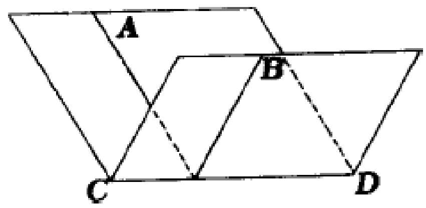
7. Learners need to be encouraged to have fun while doing constructions. In general, learners are more apprehensive when it comes to Geometry. Enjoying the constructions will help to alleviate this and if done well should also lead to a better understanding of geometry in general.
8. All teachers have learners in their classroom who think mathematics is too difficult for them and think it's all about numbers and calculations that they (feel they) are incapable of doing. "The awe-inspiring wonder of patterns, shapes, visual proofs and possibilities that could be explored in classrooms and can lead to, at the least an appreciation of, and at best a fascination with, the mathematics we see in the world around us every day." (Fran Watson, Mathematics teacher)
9. If learners find it challenging to fold accurately, they could easily become discouraged. The teacher needs be encouraging. Perhaps remind them how difficult it probably was when they learned to walk (another physical task) and how that must have resulted in mishaps (falling over) before they got it right; today they probably do it without thinking which shows how easy a skill becomes if it is practised.
10. A4 printing paper or examination pad paper are both acceptable to use. Wax paper, however is ideal because the creases are clear, and learners can see where one point meets another and where sides meet more clearly.

Perpendicular lines

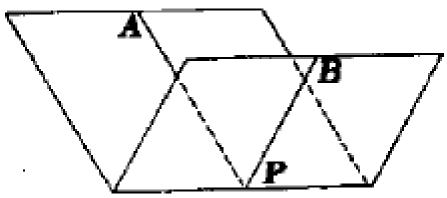
1. A line perpendicular to a given straight line
2. The perpendicular to a line at a point on the line
3. A line perpendicular to a given line passing through a given point not on the line
4. The perpendicular bisector of a given line segment

Instructions:

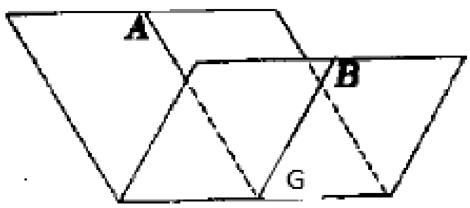
1. Draw a line segment AB.
2. Fold the page ensuring that a portion of the line segment AB folds onto itself. Make the crease.
3. Draw a line on the crease and call it CD.
4. $AB \perp CD$



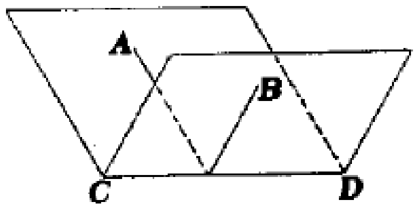
5. Using the same paper, mark a point, P, on the line AB. Fold the paper at P, ensuring that a portion of line AB folds onto itself before making the crease.
6. Draw a line on the crease. The new line is perpendicular to line AB and passes through the point P which is a point on line AB.



7. Using the same paper, turn it if necessary, so that AB is horizontal. Mark a point G anywhere above or below line AB.
8. Turn the page and fold it over so that the crease falls on point G. Mark the crease clearly, open the page and draw a line on the crease. The new line passing through G is perpendicular to Line AB.



9. Using the same paper, fold the paper so that the points A and B are on top of each other. Make a crease.
10. Draw a line on the crease and mark it CD. CD is the perpendicular bisector of AB.

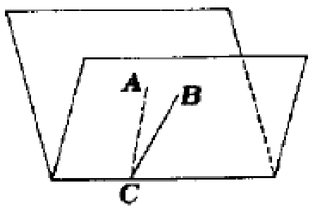


Angles

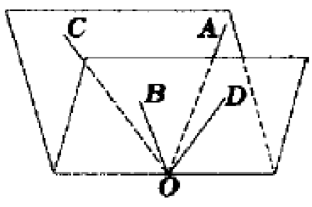
1. The bisector of a given angle
2. The formation of a right angle
3. Vertically opposite angles

Instructions

1. Using a ruler, draw line segment AB. Draw another line segment to form an angle at B. Name the other point C.
2. Fold the page so that AB and BC lie on top of each other. Mark the crease.
3. Draw a line on the crease. Name the line segment BD.
4. BD bisects $A\hat{B}C$



5. Forming a right angle was done in all 4 of the perpendicular constructions.
6. On the same piece of paper as the bisected angle, draw two line segments AB and CD. The lines must intersect. Mark the point of intersection O. (For convenience, avoid them intersecting at right angles)
7. Fold and crease the paper through vertex O, placing OB on OC. Do OA and OD also coincide?
8. Unfold the paper and repeat but this time, crease through the vertex O, placing OA on OC. Do OD and OB also coincide?



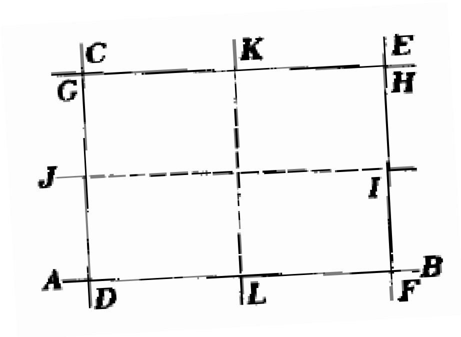
9. Repeat the instructions with a two different line segments, time permitting.

Triangles

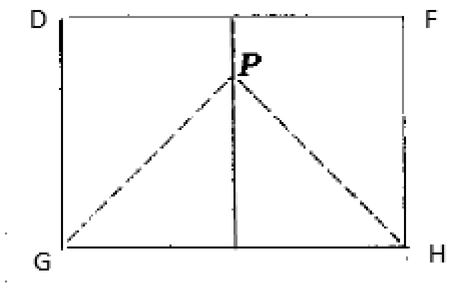
1. Isosceles triangle
2. The base angles of an isosceles triangle
3. The sum of the angles of a triangle

Instructions

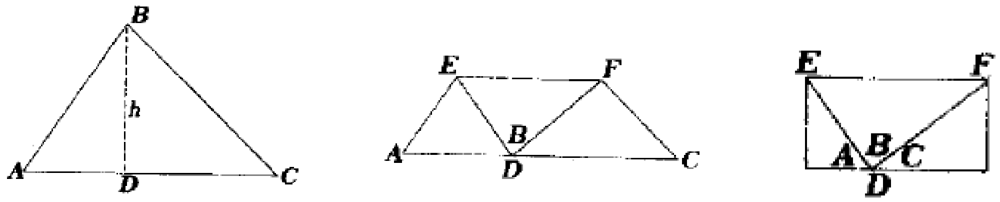
1. To form an isosceles triangle a rectangle is required. This requires the skills used in the first four constructions.
2. Fold a line on a sheet of paper and call it AB. Mark points D and F on Line AB (D very close to A and F very close to B).
3. Fold and crease a line perpendicular to AB at D and a line perpendicular to AB at F. Label the new lines CD and EF.
4. Mark a point G on line CD (very close to C). At G, fold a perpendicular line to CD. Mark the point of intersection of this line and EF, H.
5. Confirm, by folding that GH is perpendicular to EF. Rectangle DGHF will now be used to construct an isosceles triangle.



6. Fold the perpendicular bisector of DF (named KL in the diagram above). Note that this has cut the rectangle into two equal halves.
7. Choose any point on the perpendicular bisector and label it P.
8. Fold lines from P to vertices G and H.
9. Confirm by folding that 2 triangles of equal size have been formed.
10. Triangle GPH is isosceles. The folding in the previous step confirmed that the base angles of an isosceles triangle are equal.



11. Draw any triangle. Label it ABC.
12. Use previous knowledge to draw in the perpendicular height of the triangle from vertex B. (A line perpendicular to a given line passing through a given point not on the line). Label the height BD.
13. Fold the vertex B onto the base (AC) of the triangle. Fold the base angles A and C to the base to the base of the perpendicular height (D).
14. What is known about the three angles at D? (adjacent angles on a straight line and they add up to 180^o). These are the same angles in the triangle. Hence, the angles of a triangle are also equal to 180^o.
15. This construction can also show why the area of a triangle is half the area of a rectangle.



Properties of quadrilaterals

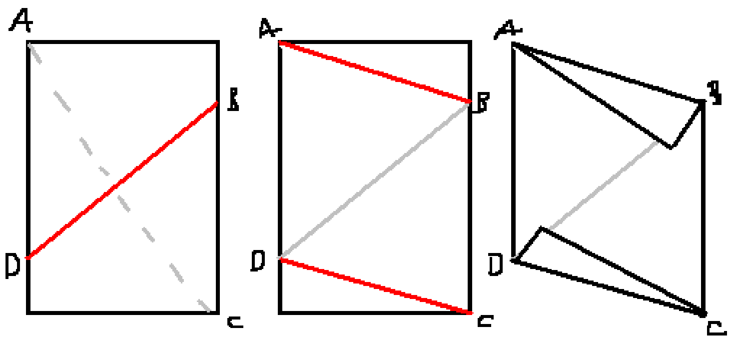
Due to time constraints we will take a brief look at how to construct a rhombus from a rectangular piece of paper to use for the investigation of some properties.

A similar exercise could be done with the rectangle, the square and the parallelogram.

Instructions:

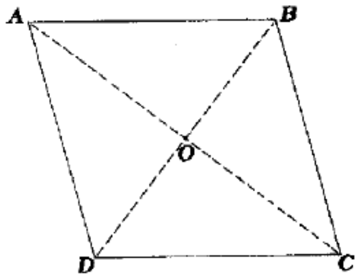
1. Fold the paper so that two opposite corners (call them A and C) meet. This creates a fold going from one long side to the other; call the points where this fold meets the long edges B and D so that A is on the same side as D and B is on the same side as C. Or see the image.
2. Unfold.
3. Fold along AB and CD.

   Image below:



4. Interestingly, this is the largest possible rhombus that can be cut/folded from any given rectangle.

5. Learners could now use this rhombus to investigate the properties. Fold and then draw in diagonals AB and CD. Mark the point of intersection O.



6. Some ideas:
   - Compare AO, BO, CO and DO by folding (diagonals bisect each other)
   - Fold along the diagonal BD. What angle is the reflection of $A\hat{B}D$ ($C\hat{B}D$ – diagonals bisect the angles. This could be repeated with the other three vertices as the focus)
   - Is ΔABD congruent to ΔCBD?

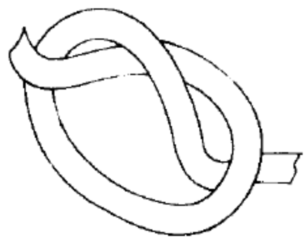
Time permitting:

Construction of Polygons by using paper and knots

These are rather interesting and would allow learners to be more aware of what regular shapes can be made with just a little paper. Perhaps a project on finding other ways to construct regular polygons would assist learners in spending more time researching and practising these skills.

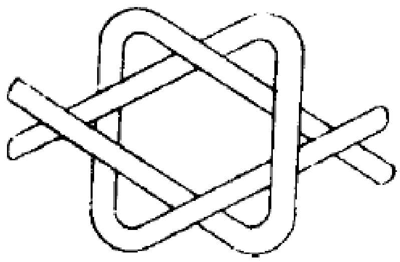
1. Pentagon

Use a long strip of paper (if an A4 sheet of paper is being used a width of 2cm would work well). Tie a knot in the paper. Gently pull the knot tight and crease it flat. Cut the excess lengths off. You have a regular pentagon.



2. Hexagon

Use two long strips of paper of equal width. Prepare a knot as shown in the diagram:



Tie the knot and pull to tighten then crease it flat. Cut off excess lengths. You have a regular hexagon.

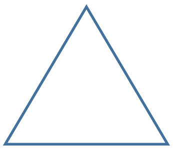
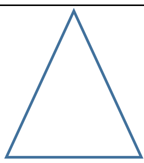
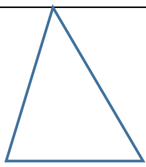
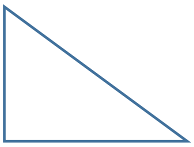
7	**2 hours**	**2-D SHAPES AND SYMMETRY (Grade 4-6)**	**Facilitator:**	**What you will need:**

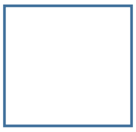
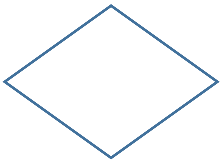
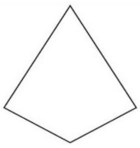
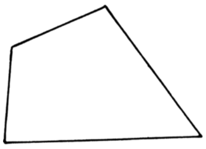
INTRODUCTION

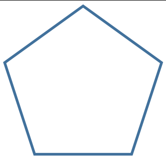
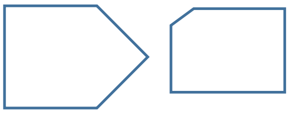
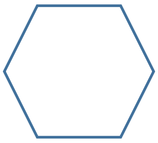
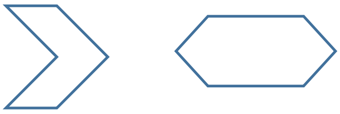
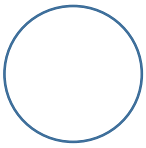
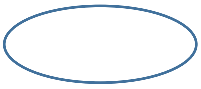
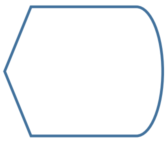
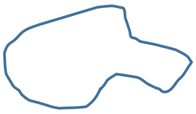
At the development age that learners have reached in Grade 4, they are able to compare 2-D shapes based on their properties. They can put two or more shapes next to each other, identify and describe the differences and the similarities.

In this activity we provide the facts sheet and participants will develop in pairs, as a class exercise, the conceptual development in a learning centred classroom.

For the benefit of concept developers, we are now going to formally compare and list similarities and differences between various shapes that learners must master in Grade 4. Please mark the listed properties on the 2-D shape. This list can also be used and extended in Grade 5 and 6 and is worth spending time on.

<table>
<tr><td>Name of shape</td><td>Diagram of shape</td><td>Properties of shape</td></tr>
<tr><td colspan="3" align="center">Triangles</td></tr>
<tr><td>Equilateral triangle</td><td></td><td>a. Three straight sides
b. All sides equal length
c. All angles same size</td></tr>
<tr><td>Isosceles triangle</td><td></td><td>a. Three straight sides
b. Two sides equal length
c. Two angles same size</td></tr>
<tr><td>Scalene triangle</td><td></td><td>a. Three straight sides</td></tr>
<tr><td>Right triangle</td><td></td><td>a. Three straight sides
b. One right angle</td></tr>
</table>

Quadrilaterals		
Square		a. Four straight sides b. All sides equal length c. All angles same size (90^o) d. Two pairs of parallel sides
Rectangle		a. Four straight sides b. All angles same size (90^o) c. Two pairs of parallel sides d. Two pairs of opposite sides equal length
Rhombus		a. Four straight sides b. All sides equal length c. Two pairs of parallel sides d. Two pairs of opposite angles same size
Parallelogram		a. Four straight sides b. Two pairs of parallel sides c. Two pairs of opposite sides equal length d. Two pairs of opposite angles same size
Trapezium		a. Four straight sides b. One pair of parallel sides
Kite		a. Four straight sides b. Two pairs of adjacent sides equal c. One pair opposite angles same size
Scalene quadrilateral		a. Four straight sides

Pentagons		
Regular pentagon		a. Five straight sides b. All sides equal length c. All angles same size
Irregular pentagon		a. Five straight sides
Hexagons		
Regular hexagons		a. Six straight sides b. All sides equal length c. All angles same size
Irregular hexagons		a. Six straight sides
2-D shapes with curved sides		
Circles		a. Closed shape with curved side b. Side at every point at equal distance from the centre
Irregular shapes with curved side		a. Closed shape with curved side
2-D shapes with curved, straight or irregular sides		
Irregular 2-D shapes with curved and straight sides		a. Closed shape with curved and straight sides
Irregular 2-D shape		b. Closed shape

Instruction for class activity:

1. Participants work in pairs.
2. Discuss and phrase one of the main concepts that can be developed about 2-D shapes.
3. Design an exercise that can serve to develop this concept.
4. Set up learner instructions for the exercise.
5. Describe the teacher's role in this exercise.
6. Describe how assessment for learning will take place during the exercise.

DOT PAPER

8a	30 minutes	Selection of topics for participant presentations	Facilitator:	What you will need:
				6 coloured boards per pair Koki pen per participant

1. Settle the participants and explain that they are now going to demonstrate the development of a concept in the topic *Symmetry* as it should be done in a learning centred classroom.
2. Remind participants that they can use the Content Booklet and the Trainers Guide as sources.
3. Explain that they will have 1 hour to plan the demonstration and 15 minutes per pair to give the demonstration with 10 minutes for discussion and feedback from the group.
4. These are the demonstration options:
    a. Line symmetry
    b. Reflection
    c. Rotational symmetry
5. It is preferable that there is more than one demonstration per topic, because the ideas of concept formation, conceptual understanding and the learning centred classroom are new and we want to pool participants' thinking and suggestions.
6. After each group has been given 15 minutes the participants will give constructive feedback. (Always start with a positive observation and then give constructive comments).
7. Thank each group for their effort.

Use this time to ensure that participants do the following:

8. Clearly and correctly explain the concept
9. Use the correct language / terminology

Also use this time to clear up any misunderstandings or misconceptions that participants may have. Participants must leave with a clear understanding of how to teach these concepts. If a lesson is demonstrated incorrectly, use this time to re-demonstrate the lesson correctly.

8b	3 hours	Lesson Demonstrations and Feedback	Facilitator:	What you will need:
				• Flipchart Paper • Markers • Improvised resources

1. Tell participants that you are really looking forward to their presentations.
2. Remind participants of these criteria explained in the briefing.
3. Remind participants that their presentations should take 15 minutes.
4. Stop the presentations after the allocated time. You must be strict with the time, otherwise not everybody will have a chance to present.
5. If a group does not manage to do very much within the time, speak to them about time management. Explain that they will not have much more time than this in class to do these presentations. Discuss how the group could speed up.
6. Ask the group to state the grade, topic and subtopic for the lesson that they will present.
7. After each lesson demonstration encourage conversation for critical and constructive feedback.
   Encourage all participants to take part in the feedback session.
   Ensure that all feedback starts with something positive – our approach is to build confidence!

9	1 hour 30 minutes	ORIENTATION TO THE TRAINER'S GUIDE	Facilitator:	What you will need:

Note: If you have any extra time, spend it on this activity, particularly points 4 and 6.

1. Settle participants with all their materials.
2. Give each participant a copy of the **Trainers Guide** and **Training Handout**.
3. Explain to participants that the **Trainers Guide** and **Training Handout** contains all the activities for the Term 1&2 training.

Planning the training session:

a. Tell participants to look carefully at the programme at the front of the trainer's guide.
b. Go through this programme and tell participants which activities to complete when training other trainers.
c. Go through this programme and tell participants which activities to complete when training teachers. (This will depend on the numbers of hours for this training)

Orientation to the guide and handout:

a. Go through each activity in the trainer's guide and look at the corresponding resources or section in the training handout.
b. Work with participants to summarise the key steps and points of each activity.
c. After you have done this for each activity, revise the order of activities, and the main points for each activity. For example:
   - Start with the **Guidelines for facilitators and participants.**
   - You have 30 minutes for this.
   - You must: tell participants to think about when real learning takes place; get them to discuss this with a partner; write a list of key points; discuss what is the same and different between a classroom and an adult training event; create a list of guidelines for facilitators and participants; ask participants to follow guidelines and commit to following facilitator guidelines.

The point of doing this is try and ensure that trainers clearly understand each activity and internalise as much of the workshop as possible.

If time allows, allocate different activities to volunteers, and ask them to present a 'dry-run' presentation of the activity. After each presentation, ask the other participants to give feedback based on the following: a. Was the activity presented correctly? b. Did the main points of the activity come across clearly? c. Did the presenter give clear instructions? d. Was the presenter audible? e. Did the presenter interact effectively with participants? f. Did the presenter manage time effectively? **Finally, thank participants for their presentations, and hold a closing discussion:** a. Ask: Which activities are you worried about presenting or facilitating? Why? b. Try to address any concerns that participants may have. c. Wish participants well for their training.

10	**1 hour**	**TRAINING OF TEACHERS: PLANNING SESSION**	**Facilitator:**	**What you will need:**
1. Explain that this is an opportunity for Coaches and Subject Advisors to work together to talk about the logistics of the teacher training sessions in their district. 2. Allow participants to sit together in groups and discuss relevant issues. 3. If all the logistics are sorted, then participants should talk about co-facilitation, and who will present which activities. 4. They should also speak about resources in their district, like data-projectors and speakers.				

11	**30 minutes**	**POST TEST**	**Facilitator:**	**What you will need:** • Copies of post test
1. Work together to hand out copies of the post-test to participants. 2. Remind participants that the purpose of these tests is to measure the success of the training, not to measure the scores of individuals. 3. Remind participants of the test conditions and available time. 4. As participants complete the test, walk around and offer practical assistance if needed. 5. Once time is up, help to collect and collate tests in an orderly fashion.				

12	30 minutes	FINAL QUESTIONS AND ANSWERS CLOSURE AND EVALUATION	Facilitator:	What you will need:

1. Settle participants so that you have their attention.
2. Remind participants that we want them to IMPLEMENT THIS TRAINING IN A MEANINGFUL WAY.
3. Ask participants to think through all the materials, content, skills and information they have engaged with in this workshop. Give them time to look through materials as they do this.
4. Next, ask participants if they have any final questions.
5. Answer each question as clearly as possible. Where appropriate, involve participants in answering.

CLOSURE AND EVALUATION

1. Settle participants so that you have their attention.
2. Show participants the video: 'Teaching is tiring – but worth it'
   https://www.youtube.com/watch?v=ZSjIzi0SkjU
3. Give participants an evaluation form, briefly take them through the form, and then ask them to please complete it thoughtfully and carefully.
4. Collect the completed evaluation forms.
5. Call participants to attention and ask them to share some of the positives that they take away from this training. This can be absolutely anything: new content that they have learned or clarified; a new skill; a better understanding of the curriculum; new enthusiasm for their job; a closer working relationship with a colleague; etc.
6. Document what participants say for your report.
7. Thank the participants for their ongoing commitment to education, and to the development of South Africa and wish participants well for their own training.

Thank you for your ongoing dedication and commitment to this cause.