Grade 7

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 4

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

Contents

INTRODUCTION: THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which influence whether a child is ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract (Piaget, 1969), was refined (Miller & Mercer, 1993) to include a middle stage, namely the "concrete-representational-abstract" stages. This classification is a handy tool for mathematics teaching. We do not need to force all topics to follow this sequence exactly, but at the primary level it is especially valuable to establish new concepts following this order.
- This classification gives a tool in the hands of the teacher, not only to develop children's mathematical thinking, but also to fall back to a previous phase if the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may gradually pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phases.

How can this approach be implemented practically?

The table on page 7 illustrates how a developmental approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the topics, suggestions are made to implement this principle in the classroom:

- Where applicable, we suggest an initial concrete way of teaching and learning a concept and we provide educational resources at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- Where applicable, we provide pictures (representational/semi-concrete) and/or diagrams (representational/semi-abstract). It may be placed at the clarification of terminology section, within the topic itself or at the end of the topic as an educational resource.
- In all cases we provide the symbolic (abstract) way of teaching and learning the concept, since this is, developmentally speaking, where we primarily aim to be for learners to master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest that teachers present mathematics topics in three forms to provide for all learners' learning styles and preferences. They (a) explain the idea by speaking about a topic, (b) illustrate it by showing pictures or diagrams and finally (c) present the idea by symbolising it in numbers and mathematical symbols.
- Teaching in more than one form (multi-modal teaching) includes hearing the same mathematical idea in spoken words (auditory mode), seeing it in a picture or a diagram (visual mode) and writing it in a mathematical way (symbolic mode).
- Learners differ in the way they learn and understand mathematical ideas. For one learner it is easier to understand through hearing and for the other through seeing. That is why we open both pathways to the symbolic mode – because here they do not have a choice, they all have to reach that mode, be it through hearing or seeing.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the topics at the Senior Phase, we suggest ways to apply this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "speak it" or auditory mode.
- The pictures and diagrams give suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths, we teach concepts systematically. If we teach concepts out of that order, it can lead to difficulties in grasping concepts.
- Systematic teaching according to CAPS builds progressive understanding and skills.
- In turn, this builds confidence in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- We have to continuously review and reinforce previously learned skills and concepts.
- If learners link new topics to previous knowledge (past), understand the reasons why they learn a topic (present) and know how they will use the knowledge in their lives ahead (future), it can help to motivate them and to remove many barriers to learning.

How can this approach be implemented practically?

If a few learners in your class are not grasping a concept, you as the teacher need to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again, however this could cause difficulties in trying to keep on track and complete the curriculum in time.

To finish the year's work within the required time and to also revise topics, we suggest:

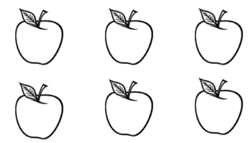
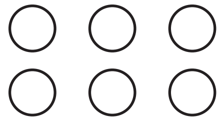
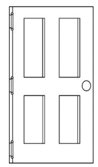
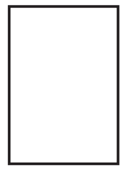
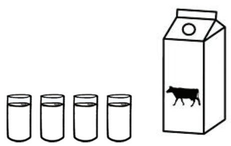
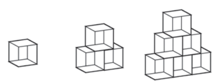
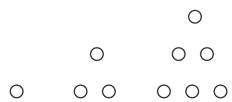
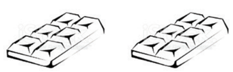
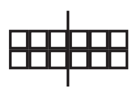
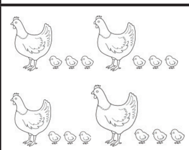
- Using some of the time of topics with a more generous time allocation, to assist learners to form a deeper understanding of a concept, but also to catch up on time missed due to remediating and re-teaching of a previous topic.
- Giving out revision work to learners a week or two prior to the start of a new topic. For example, in Grade 8, before you are teaching Data Handling, you give learners a homework worksheet on basic skills from data handling as covered in Grade 7, to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there are two parts that detail

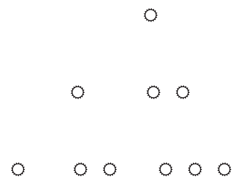
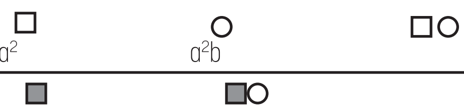
- The SEQUENTIAL TEACHING TABLE lays out the knowledge and skills covered in the previous grade, in the current grade and in the next grade.
- The LOOKING BACK and LOOKING FORWARD summarises the relevant knowledge and skills that were covered in the previous grade or phase and that will be developed in the next grade or phase.

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	**Real or physical For example:**	**Picture**	**Diagram**	**Number (with or without unit)**	**Calculation or operation, general form, rule, formulae**
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ or $2+2+2=6$ or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160 = 550cm$ Area: $L \times W = 195 \times 80 = 15\,600cm^2 = 1.56m^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250ml = 1\,000ml = 1$ *litre* *or* 1 *litre* $\div 4 = 0.25$ *litre*
Patterns	Building blocks			1: 3: 6...	$\frac{n\,(n+1)}{2}$
Fraction	Chocolate bar			6 12	$\frac{6}{12} = \frac{1}{2}$ or $\frac{1}{2}$ of $12 = 6$
Ratio	Hens and chickens		* *** * *** * *** * ***	4:12	$4 : 12 = 1 : 3$ Of 52 fowls ¼ are hens and ¾ are chickens, ie 13 hens, 39 chickens
Mass	A block of margarine			500g	$500g = 0{,}5\ kg$ or calculations like 2 ½ blocks = 1,25kg

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	SPEAK IT - explain	SHOW IT - embody	SYMBOL IT - enable
	• To introduce terminology • To support auditory learning • To link mathematics to real life	• To help storing and retrieving ideas • To help visual learning • To condense information to one image	• To promote mathematical thinking • To convert situations to mathematics • To enable calculations
IP: Geometric patterns	"If shapes grow or shrink in the same way each time, it forms a geometric pattern or sequence. We can find the rule of change and describe it in words. If there is a property in the shapes that we can count, each term of the sequence has a number value" "You will be asked to draw the next term of the pattern, or to say how a certain term of the pattern would look. You may also be given a number value and you may be asked, which term of the pattern has this value?**"**	 T1 T2 T3 T4 Question: (a) Draw the next term in this pattern. (b) Describe this pattern. What is the value of the 9th term of this pattern (T9)? **Which term has a value of 120?** To draw up to the 9th term of the sequence and to find out which term has a value of 120, is slow. One is now almost forced to deal with this problem in a symbolic way.	Say out loud: 1; 3; 6… 1; 3; 6; 10… 1; 3; 6;10;15 Inspect the number values of terms: T1: 1 = 1 T2: 3 = 1+2 T3: 6 = 1+2+3 T4: 10 = 1+2+3+4 T9: 45 = 1+2+3+4+5+6+7+8+9 General rule: The value of term n is the sum of n number of consecutive numbers, starting at 1.
SP: Grouping the terms of an algebraic expression	"We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to imagine the following pictures in your mind:"	Although not in a real picture, we can paint a mind picture to help us understand the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a^2) • Basket with yellow apples and green pears (a^2b) Or in diagram form  a b ab a^2 a^2b	Group and simplify the following expression: $4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$ $= - 3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$

TOPIC 1: INTEGERS

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the content area, 'Numbers, Operations and Relationships' and counts for 30 % in the final exam.
- The unit introduces the concept of negative numbers and also starts the development of an understanding for the continuity of the number system
- It is important to ensure that learners have a clear understanding and an adequate ability to perform all the required calculations with whole numbers. Learners need to perform the same operations as they have been so the strategies must be well developed.
- Real life applications must be used in examples to ensure a clear understanding of the role that integers have in the world.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 6	GRADE 7	GRADE 8 /FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Recognise and use the commutative; associative; distributive properties of whole numbers • Recognise and use 0 in terms of its additive property (identity element for addition) • Recognise and use 1 in terms of its multiplicative property (identity element for multiplication) • Perform calculations using all four operations on whole numbers, estimating and using calculators where appropriate	• Counting, ordering and comparing integers • Calculations with integers limited to addition and subtraction using integers. • Recognise and use the commutative and associative properties of addition with integers • Solve problems in context using the addition and subtraction of integers.	• Multiplication and division of integers. • Perform a variety of calculations with integers extended to cubes and cube roots as well as squares and square roots. • Recognise and use additive inverses for integers. • Solving problems with multiple operations using integers. • The knowledge of integers extends to Functions and Algebraic contexts later in the FET phase.

GLOSSARY OF TERMS

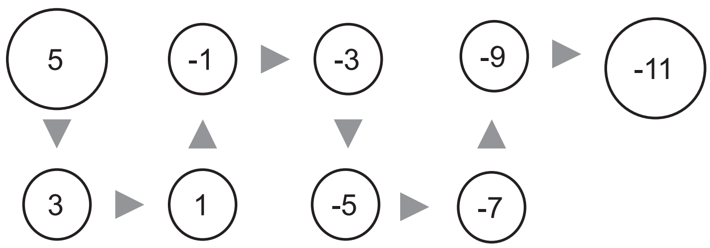
Term	Explanation / Diagram
Integers	Numbers that include negative numbers, positive numbers and zero. Fractions are NOT part of the set.
Negative Integers	All the integers less than zero
Positive Integers	Whole numbers that are greater than zero
Ascending Order	Arranging numbers from smallest to biggest
Descending Order	Arranging numbers from biggest to smallest
Additive inverse	The number that should be added to another number to get a result of zero. Additive inverses are exactly the same distance away from zero on the number line, for example 3 and -3

SUMMARY OF KEY CONCEPTS

Counting, ordering and comparing integers

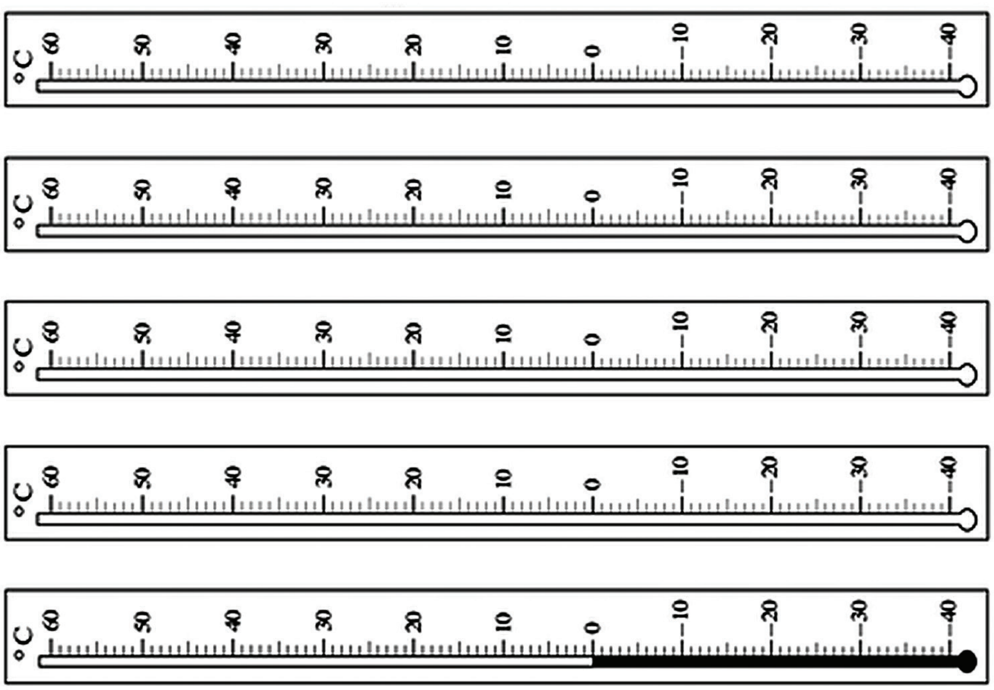
1. Learners are introduced to the continuity of the number system as the topic explains that the number system has a negative component.
2. Zero is an integer that is neither positive nor negative.
3. There are many ways to practice counting and it is very important to have learners count in a variety of intervals using integers.
4. Counting can be done verbally, but should not be limited to this form. Learners must be expected to practice counting using chain diagrams and structured, semi-structured and even empty number lines.

   In the diagram below learners should recognise that two is being subtracted each time and the sequence starts at 5. Ask learners to continue the pattern.



Using real examples would help learners identify the value of integers

Using the thermometer, you could ask learners to colour up to a particular value that you give them, then they could check each other's work and confirm they understood the value correctly.



5. Learners must be able to count forwards and backwards using integers.
6. They must also be able to order integers using various notations such as arranging the numbers in ascending or descending order.
7. Integers should also be used in exercises where learners are expected to complete sequences, number grids, number lines. Inequalities also need to be included.

   Examples:

   Write the next three numbers:
   - 9 ; 7 ; 5 ; 3 ; ......
   - -6 ; -4 ; -2 ; .....

   Fill in <, > or =:
   - 20 ______ -40
   - -320 _______ -321

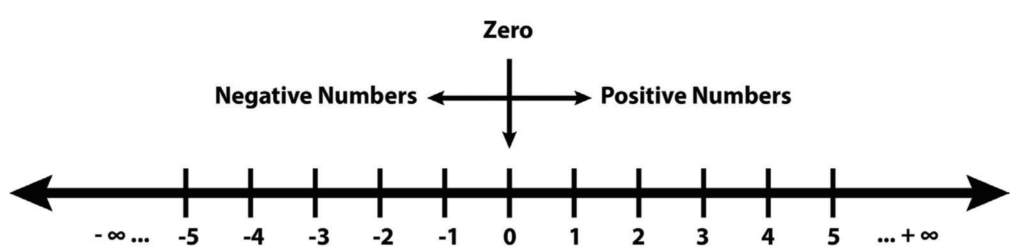
   Solutions:
   - 1 ; -1 ; -3
   - 0 ; 2 ; 4
   - >
   - >

   Learners must understand that the closer the negative number is to the zero the greater the value of the number.

Teaching Tip:
Ensuring that learners have access to a number line that depicts the number line as a continuous structure such as the one below will help them understand the position and the value of negative integers.

Sometimes a vertical numberline will assist learners in seeing more clearly when a number is larger because it will be above the number it is being compared to.

NUMBER LINE



Calculations with integers

1. Always start calculations with integers in small number ranges

Example:

$11 - 7 = 4$ *or* $4 - 7 = -3$

2. Develop an understanding in learners that subtracting an integer is the same as adding the additive inverse. of that number.

Example:

$8 - 5$	$-8 - 5$
$= 8 + (-5)$	$= -8 + (-5)$
$= 3$	$= -13$

Teaching Tip:
The use of brackets around the additive inverse is helpful especially as learners are still developing a clear understanding of integers. It is also useful to explain to students that the sign in front of a number indicates direction. Negative (-) would indicate a move to the left of the required number of spaces and positive (+) (or no sign) would indicate a move to the right.

In the beginning this can be very confusing for learners. Below is a possible method to use to help explain the confusing issue of subtracting a negative or adding a negative and so on. It can be fun for the learners too as it involves them getting out of their desks and making the movements themselves. If there are too many learners to make this possible, ask for a volunteer or two and demonstrate in front of the class.

Imagine standing up and taking a few steps in a certain direction. Let's say you take 8 steps forward. This could be represented as 0+8. 0 would be the starting point, + means you should move **forward** and 8 means you will be 8 steps from where you started.

Keep facing forward. Take 6 more steps. This would be represented as 8+6=14. Because it's a + and you moved **forward**, you are now 14 steps away from where you started.

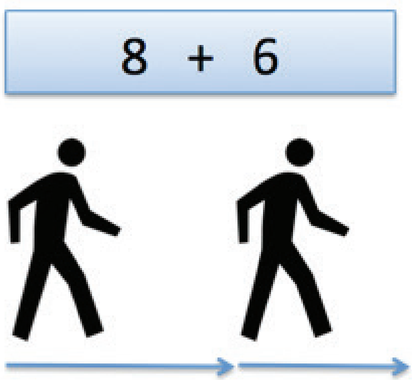
Imagine being back at the 8-step mark. Instead of moving forwards, you decide to move backwards 6 steps. This would be represented as 8 - 6 = 2. 8 being the starting point, - meaning you are walking **backwards** taking 6 steps.

To sum up before moving on, when you are walking forwards (so you can see where you're going) it is representing the addition. When you are walking backwards (so can't see where you're going) it is representing subtraction.

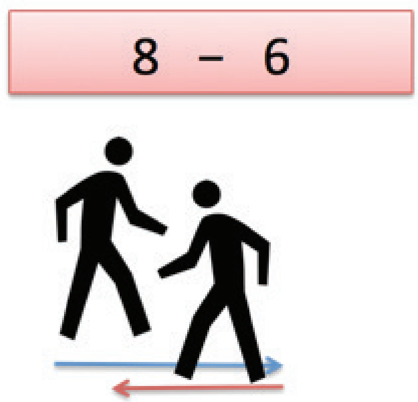
Imagine being back at the 8-step mark. What if you faced backwards and walked backwards 6 steps? So, you turned around to look at where you started but walked backwards, moving further away from the starting point. Now you are 14 steps away from where you started. Turning and facing backwards implies subtraction, moving backwards is also subtraction.

This can be represented as 8-(-6), which we know should be 14. Subtracting a negative is the same as adding!

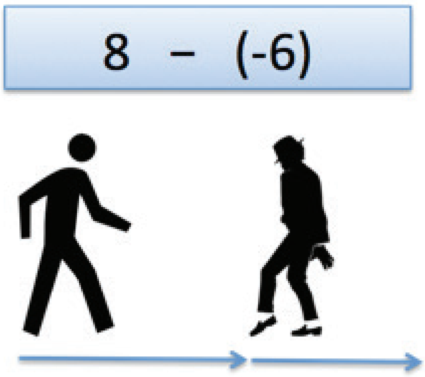
Summary: When there are 2 signs, the first sign tells us which way to face and the next sign tells us to move forwards or backwards (regardless of which way we are facing).

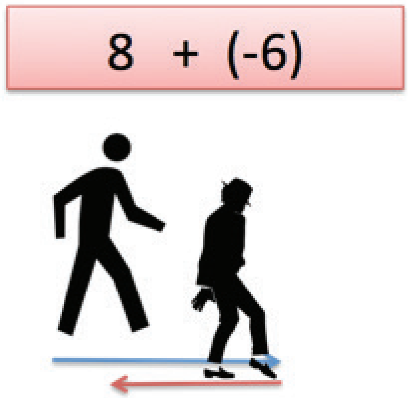


This is the easy one. Start at 8, add 6
=14
+(+)
+ → face forwards + → move forwards



This is the same as 8 subtract positive 6.
-(+)
- → face backwards + → move forwards





This is: 8 add to negative 6
+(-)
+ → face forwards - → move backwards

This is: 8 subtract negative 6
-(-)
- → face backwards - → move backwards

Properties of integers

1. Learners will notice that the commutative property applies to integers.

   Example:

   $12 + (-3) = 9$
   $(-3) + 12 = 9$
   Learners will see that they can use subtraction to check addition or addition to check subtraction. This will encourage investigation of the additive inverse property of integers and gives learners a clear understanding that enables the application of strategies even when working with much larger numbers.

2. Example:

   $-7 + 2 = -5$
   $-5 - 2 = -7$

3. Learners will notice that the associative property applies to integers.
   Example:

   $[(-3) + 2] + 5 = 4$
   $(-3) + [2 + 5] = 4$

Remind learners that the associative property means we can associate the numbers in any way we wish.

Solving problems in context using integers

1. Problems involving temperature and money make good examples when working with integers in context.

   Examples:
   If the minimum temperature on a cold morning was -3°C and the temperature increased by 7 degrees by 10 am. What would the thermometer show the temperature was?

   4°C

   James delivered newspapers for two Saturdays. He got paid R 58 for the first day and R 35 for the second day. If he owes his dad R 150 for a pair of soccer boots he lost, how much money does he still need to earn?

   (He needs R57)

2. Encourage learners to accept that a negative answer is a possibility.

TOPIC 2: NUMERIC AND GEOMETRIC PATTERNS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the content area, 'Patterns, Functions and Algebra' and counts for 25 % in the final exam.
- The unit requires revision of the concepts concerning Numeric and Geometric Patterns with an extension that requires an understanding of the effect that operations will have on integers or that patterns can result in integers.
- It is important to ensure that learners can perform all the necessary operations such as determining the next terms of a sequence or determining the rule that gives the required sequence. These basic concepts are the required foundations for a concise understanding of Algebraic concepts and Functions later in the FET phase.
- Although learners have been working with numbers and their various properties as well as with simpler versions of number patterns it is vital that learners start to expand their understanding of the formulae or rules that form the relationships within a sequence.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 6	GRADE 7	GRADE 8 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Using flow diagrams to demonstrate inverse operations, associative property and multiplication strategies. • Multiple operations performed using flow diagrams. • Extending geometric patterns or sequences with a constant difference or ratio • Determining the rule of a geometric pattern • Representing patterns in a variety of ways	• Describe and extend numeric and geometric patterns • Extend patterns that have neither a constant difference nor constant ratio • Use rules to extend patterns • Use tables with position in sequence and term values to determine the rule or pattern • Extend the use of these patterns to include integers.	• Describing the relationships in numeric and geometric patterns in algebraic terms. • Describing the general rule in a pattern in algebraic terms and gradually reducing the option for description of the rule in own terms. • Determining the terms in a sequence using equations instead of flow diagrams • In grade 9 learners will represent relationships graphically.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Patterns	A pattern may be either something that is to serve as a model to be copied or more usually a set of objects or elements arranged in order according to a rule.
Number Patterns	A list of numbers formed according to a mathematical rule. It can also be called a number sequence.
Consecutive terms	Terms that follow each other according to the rule of the sequence.
Rules	The explanation of how the pattern is arranged.
Term	A number or the combination of a number and a variable in a numerical pattern or mathematical expression.
Constant Difference	Every time we can add or subtract the same number to a previous term to result in the next term this pattern has a constant difference.
Constant Ratio	Every time we can multiply or divide by the same number to alter a previous term to result in the next term this pattern has a constant ratio.
Geometric Patterns	These are patterns that are represented by diagrams. Each diagram is altered to the next by applying a constant rule.

SUMMARY OF KEY CONCEPTS

Numeric Patterns with a constant difference or ratio

1. Recognising and extending a numeric pattern.

Teaching Tip:
Learners have been working with this concept throughout the intermediate phase. They have continuously described patterns and extended them by using their own language and not bound by any conventional methods. In Grade 7, the idea is to start extending the learners to use conventional, algebraic language and methods to perform the same task. Learners must now extend the idea further by using integers in the pattern and working with patterns that would have input, output or a variable that has a negative integer.

2. Learners must be making use of correct terminology such as constant difference and constant ratio when describing the patterns and must refer to each portion of the pattern as a term.
3. Learners must be able to extend a numeric pattern as well as determine a missing term within a numeric sequence.

Examples:
-8; -12; -16; -20… Constant difference of (-4)
2; -4; 8; -16; 32…Constant ratio of (-2)

Teaching Tip:
Sequential teaching of this section is vital. It is therefore an excellent idea to start with patterns that have a constant difference or ratio that would have been covered in Term 3 and then showing how the same principle can be applied when integers are used in the pattern.

Although the concept of patterns has been covered previously it is wise to use pictures and concrete examples as patterns are explained and to ensure that learners develop a concise understanding of the influence on the pattern when applied to integers.

Using tables to investigate and extend numeric patterns

1. Using tables can be very useful when learners must identify and extend numeric patterns. Learners must be encouraged to determine the result when a rule is applied. The rules used in Term 4 must include negative integers as this is the extension required to prepare learners for further study in Mathematics.

Example
Find the 20th term of the following pattern, given the following rule:
multiply by -2 and subtract 4
$\times (-2)\ -4$

Position in sequence	1	2	3	4	20
$\times (-2) - 4$	-6	-8	-10	-12	?

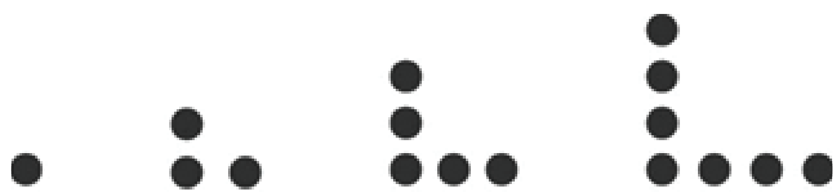
20 x (-2) -4 = -40 - 4 = -44

Learners should not just be finding consecutive terms and must use tables and rules to determine a required term.

Geometric Patterns

1. Geometric patterns do not need to be applied to integers but it is necessary to revise the required techniques when working with these types of patterns.
2. Geometric Patterns are numeric patterns represented diagrammatically. The diagram represents the structure of the number pattern so learners must be able in relate the visual (diagram) to the abstract (rule)
3. Learners must be able to draw the next diagram in the sequence and they must be able to represent the diagram as a numeric sequence.

   For Example:



There are many other patterns such as the example given above. Learners must be able to draw the next pattern in the sequence and they must also understand that this must be written as a numeric sequence 1 ; 3 ; 5 ; 7 ...

Teaching Tip:

Learners can take some time visually representing a pattern using matches or small sweets (jelly tots). Remember that we are moving to the abstract, but should a learner be struggling to see these relations then backtracking to the visual will help them see the pattern. Practice will help learners see the patterns develop. Keeping these lessons practical will also aid learners in their understanding of the concepts.

TOPIC 3: FUNCTIONS AND RELATIONSHIPS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the content area, 'Patterns, Functions and Algebra' and counts for 25 % in the final exam.
- The unit revises and extends the identification, description and extension of a variety of number patterns and is an essential element required in the clear understanding of Algebraic conceptual knowledge.
- It is important to ensure that learners can perform all the necessary operations such as determining inputs, outputs and rules. These basic concepts are the required foundations for a concise understanding of Algebraic concepts and Functions later in the FET phase.
- Although learners have been working with basic flow diagrams and tables of values, it is vital that learners start to expand their understanding of the formulae or rules that form the relationships between input and output values and that learners can determine inputs and outputs that include integers.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 6	GRADE 7	GRADE 8 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Using flow diagrams to demonstrate inverse operations, associative property and multiplication strategies. • Introduction to finding the input, output and rule of number sequences	• Determining input, output values or rules • Use tables with input and output values to determine the rule • Determine, interpret and justify equivalence or varied representations of the same relationship or rule. • Expanding the applications for functions to include those where input or output values include integers and also using integers in the rule.	• Describing the general rule in a patterns in algebraic terms and gradually reducing the option for description of the rule in own terms. • Determining input and output values using equations instead of flow diagrams • In grade 9 learners will represent relationships graphically.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Input	The starting value or the independent value in a flow diagram
Output	The final value or the dependent value in the flow diagram. It is the value obtained after the rule has been applied to the input value.
Process or Rule	Process or Rule is what is done to the input value that results in the output value. These can be a single operation or a variety of operations applied to the input value.
Flow Diagram	A diagram that shows the input, process and the result or output value diagrammatically. That is it is a diagrammatical representation of a number sentence.

SUMMARY OF KEY CONCEPTS

Using flow diagrams to determine inputs, outputs and rules or processes

1. This is an extension of the content that was covered in Term 2 and 3. In this section integers are used in the process and either inputs or outputs can be determined.

Teaching Tip:
Learners have been working with this concept in previous Grades and earlier in the year. Start the lesson by giving examples of the work covered previously where inputs and outputs were whole numbers only and gradually increase the level of difficulty to include integers at different stages of the process.

2. Learners must be capable of finding the input and output values if they are given the rule or process and they can be expected to find both in the same question.

Example:

Input values		RULE		Output values
3	→		→	-3
x	→	$\times(-2)+3$	→	-5
10	→		→	z
y	→		→	13

Remind learners that when they are finding the input value (given the rule and the output value), inverse operations are required.

$x = 4$

$y = -5$

$z = -17$

Teaching Tip:
Encourage learners to start using symbols to represent the unknown values in a flow diagram this is going to form the starting concepts for the next section ‘Algebraic Expressions’

Equivalent forms representing input, process and output

1. Learners must be able to recognise the same information represented in a variety of ways.

   It is important that learners recognise functions in any of these formats.

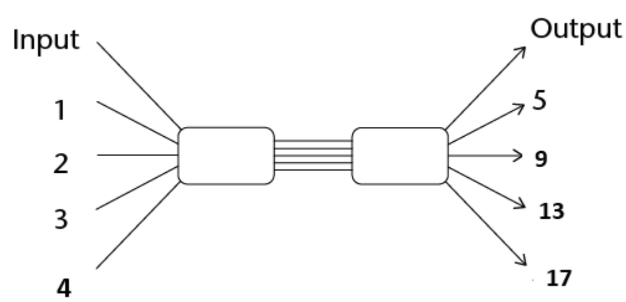
   For example:
   Using natural numbers from 1 to 4 as the input and the rule, multiply by 4 then add 1, complete the following table:

Input				
Output				

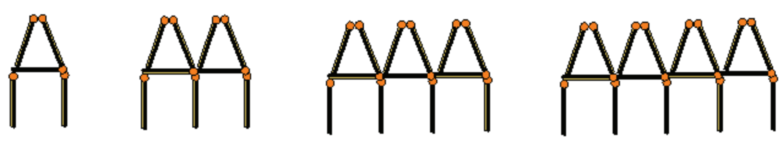
   Using the function $y = 4x + 1$, complete the following table

x	1	2	3	4
y				

   Find the rule in the following flow chart



The following pattern is made with matchsticks:



Find the general term to represent the number of matches used in each pattern.

Note: ALL four of the above examples represent EXACTLY the same information.

Teaching Tip:
Have learners draw a comparative chart in their books where examples of a flow diagram given can be represented in a variety of ways such as in a table or even graphically (this would be done later in the senior and FET phase)

Use this opportunity to remind learners of when they used complex flow charts in Term 2 and 3 where more than one operation was performed on the input to result in the output.

2. The focus in this term must be to alter the method of thought from concrete number sentences to the more abstract representation using algebraic terminology.
3. Tables should have been used extensively in the intermediate phase so it is important to emphasise the fact that representations in various forms are equivalent.

TOPIC 4: ALGEBRAIC EXPRESSIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the content area, 'Patterns, Functions and Algebra' and counts for 25 % in the final exam.
- The unit revises the Algebraic Basics covered in Term 3 and introduces learners to the use of integers as coefficients to the variable and is an essential element required in the clear understanding of Algebraic conceptual knowledge.
- It is important to ensure that learners understand mathematical language and how to replace unknown values with a variable. These basic concepts are the required foundations for a concise understanding of Algebraic concepts, equations and functions later in the FET phase.
- Although learners have been working with number sentences it is vital that they progress in understanding algebraic purposes in setting a general solution for a problem that may need to be solved or simplified.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 6	GRADE 7	GRADE 8 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Number sentences as an equivalent form of flow diagrams and tables as used to determine input and output values. • Write number sentences to describe problem situations	• Using Mathematical language with a focus on algebraic language • Recognize and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae and equations. • Using integers when writing an expression or when it is suitable for the solution to a problem in context.	• Recognise and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae and/or equations • Recognise and identify conventions for writing algebraic expressions • Identify and classify like and unlike terms in algebraic expressions • Recognise and identify coefficients and exponents in algebraic expressions

GLOSSARY OF TERMS

Term	Explanation / Diagram
Algebra	The branch of mathematics that deals with generalised arithmetic by using letters or symbols to represent numbers.
Expression	A mathematical statement which can include variables, constants and operations, for example 3a + 4
Symbol	Replaces something else. Examples include a + that replaces the word add.
Variable	This is a symbol (usually a letter) that may take the place of a value from a range of possible values. For example, in x + 3, x is the variable
Coefficient	A constant by which a variable is multplied. For example 5x, 5 is the coefficient. However, a variable could be representing the constant. For example, for ax, a is the coefficient of x
Constant	This is a value that remains unchanged and does not contain a variable in the term. For example, in x + 3, +3 is the constant
Term	Part of an algebraic expression, each term is separated by a + or - sign

SUMMARY OF KEY CONCEPTS

Recognise and interpret rules or relationships represented in symbolic form

1. Learners have previously worked with Algebraic Expressions in Term 3. Learners must be extended this term and must use integers as coefficients and constant terms and understand the value they bring to Mathematics.

Teaching Tip:
Remind learners of the work covered when they were finding the rule for number patterns at the beginning of Term 4 as well as the strategies that were used in Term 3 for the writing of algebraic expressions. By this stage it should be becoming more clear to learners that there is a connection between patterns, flow diagrams and algebra. It will be helpful to start by going back to this concept and using it as a platform to introduce the idea of a variable and integer combination in an expression.

2. Learners must at this stage be comfortable with replacing the older symbolic descriptors for an unknown amount with recognised symbols or variables when given words that need to be converted into mathematical language.

Example:
$-4 \square + 3$ (The acceptable notation used in previous grades, particularly the foundation phase) must be converted into an acceptable algebraic general rule $-4x + 3$.

Teaching Tip:
Remind learners that this was introduced in Term 3 and that the only "new" concept is the use of integers

3. Learners must be able to set up an expression by replacing words with acceptable symbols and by replacing outstanding balances with negative integers.

Identify variables and constants in given expression

1. Learners must be able to identify the components of an algebraic expression. This means they must know what each component is called and how to identify that component.

Example: $\boxed{-2y + 5}$ Algebraic Expression

Coefficient (integer) Variable Constant (integer)

This expression is made up of two terms.

Ensure learners can change verbal information into algebra.

For example:
If I took a certain number and multiplied it by negative 5 then added 10 to it, what would this expression look like?

$(-5a + 10)$

Do a number of these with learners until they are confident in their skills. Remind them that any letter of the alphabet could be used as an unknown except 'o' as it can easily be mistaken for a zero.

Once learners have completed an expression you can ask them questions such as:

- What is the coefficient?
- What variable did you use?
- What is the constant term?
- How many terms do you have?

For the expression created above, the answers would be:
-5 ; a ; 10 ; 2

TOPIC 5: ALGEBRAIC EQUATIONS

INTRODUCTION

- This unit runs for 4 hours.
- It is part of the content area, ‘Patterns, Functions and Algebra’ and counts for 25 % in the final exam.
- The unit revises and extends the work from Term 3 on equations and is an essential element required in the clear understanding of Algebraic conceptual knowledge.
- It is important to ensure that learners clearly understand mathematical language and how to replace unknown values with a variable. These basic concepts are the required foundations for a concise understanding of Algebraic concepts, equations and functions later in the FET phase. Learners are required to solve for the unknown.
- Although learners have been working with number sentences, equations require a clear understanding of number sentences and inverse operations to solve for an unknown value that satisfies a problem. This term the focus is on equations that include integer solutions and components.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 6	GRADE 7	GRADE 8 /FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Write number sentences to describe problem situations • Construction of algebraic expressions based on a given problem.	• Write number sentences to describe problem situations • Analyse and interpret number sentences that describe a given situation • Solve and complete number sentences by: -- inspection -- trial and improvement • Identify variables and constants in given formulae or equations • Determine the numerical value of an expression by substitution. • Determining a solution that could include an integer.	• Solving linear equations in a variety of contexts including algebra, trigonometry, geometry and calculus. • Using substitution to determine values in financial mathematics • Setting up and solving advances equations relating to a variety of word problems • The solution of simultaneous equations to determine intersection of functions.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Equation	The formal word for a number sentence. A mathematical statement including an equal sign
Symbol	Replaces something else. Examples include a + that replaces the word add.
Variable	This is a symbol (usually a letter) that may take the place of a value from a range of possible values. For example, in $x + 3$, x is the variable
Coefficient	A constant by which a variable is multplied. For example $5x$, 5 is the coefficient. However, a variable could be representing the constant. For example, for ax, a is the coefficient of x
Constant	This is a value that remains unchanged and can be a positive or negative integer. For example, in $x + 3$, $+3$ is the constant
Term	Part of an algebraic expression, each term is separated by a + or - sign
Substitution	Replacing the unknown with a calculated or given amount to determine the value of the expression

SUMMARY OF KEY CONCEPTS

Writing, analysing and interpreting an equation

1. Learners have been working with number sentences derived from input/output diagrams in the work covered earlier in the year and during the course of Term 3 when they worked with number patterns and algebraic equations.

Teaching Tip:
Learners have been writing number sentences using input and output diagrams throughout the intermediate phase. They must understand that they are performing the same task in a different context. The only extension is that they could expect a negative result or a negative coefficient in the equation.

2. Learners already been shown that they can replace unknown values with a variable. They have also written algebraic expressions and have identified the parts of these expressions.
3. Learners have solved basic equations that have resulted in whole number answers during the course of Term 3.
4. Using a table to show the differences between number sentences and algebraic equations is a good method to ensure that knowledge transfer is concise.
5. Learners must practise changing number sentences into algebraic equations, changing word problems into algebraic equations and describing an algebraic equation in their own words.
6. The more learners practise this concept the easier it will become.
7. The only extension for Term 4 is to include examples that contain or result in integers answers.

Solving and completing equations

1. Learners must be able to solve and complete equations using a combination of two methods:
   - Inspection: which means solving by considering the information given and finding the solution through logic and a mental calculation.
   - Trial and improvement: this means to try a value and check if it makes the statement true - in other words, does the left hand side equal the right hand side when the value chosen is used. If the first 'trial' doesn't work, choose another variable and try to improve on what was chosen before until the value chosen makes the left hand side equal to the right hand side.

Teaching Tip:
Learners must always check to see if their answers make sense in the context of the question. This will be a good skill if applied later in the senior phase, when dealing with financial mathematics and even geometry.

Example: Solve $5x + 2 = -18$
Learners would “guess” -3 (This is a trial which will need to be improved upon)
$(5 \times -3) + 2 = -13 \quad LHS \neq RHS$
Learners now try -4 (This is an improvement as it gives an accurate result)
$(5 \times -4) + 2 = -18 \quad LHS = RHS$

Finding the value of an expression by means of substitution

1. Learners have been substituting values into rules as they worked with input and output diagrams, tables of values and even when calculating area and perimeter throughout the intermediate phase. This is merely the formalisation of their understanding.

2. Example:

   What number must be multiplied by 20 to result in -120?
   Learners would by inspection determine the answer is -6.
   The check is done by substituting -6 into the multiplication and determining if the result is correct. Thus if $x = -6$ then $20 \times -6 = 120$
3. This can be extended to solving equations using real life context such as temperature and money where negative results are justifiable.

4. Example:

   Justine has been looking for her lost hair clips. She has 7 in her hair. She finds 10 in the tin on her dresser and 13 under the couch in the lounge. If she had 50 at the beginning of the year, how many does she still need to find?

   If this is structured correctly the negative result will indicate the number of items still outstanding.
   $7 + 10 + 13 - 50 = x$
   $30 - 50 = -20$
   She still needs to find 20 hair clips.

TOPIC 6: DATA HANDLING

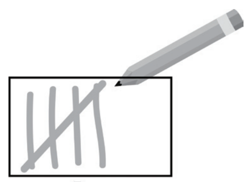
INTRODUCTION

- This unit runs for 10,5 hours, which includes: Collect, organize and summarize data (4 hours); Represent data (3 hours) and Interpret, analyse and report data (3,5 hours)
- It counts for 10% in the final exam.
- The unit covers all aspects of data handling as laid out above
- It is important to note that learners need to be exposed to a variety of contexts that deal with social and environmental issues. Learners also need to practice collecting, organising, representing and analysing data. Time needs to be spent discussing and showing learners the differences between the different types of graphs and when one may be more useful than the other.
- If you have access to computers, the learners could use them to draw some of the graphs in Excel.
- The purpose of teaching data handling is to provide learners with the knowledge of how data is collected and the ways in which it can be represented.
- Surveys, graphs and charts are often used by the media to inform, persuade and at times, mislead the audience. Learners need to be made aware of this.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 6	GRADE 7	GRADE 8 / FET PHASE
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Using simple questionnaires (yes/ no type response) • Draw a variety of graphs to display and interpret data including: double bar graphs • Analyse data by answering questions related to: central tendencies – (mode and median • Examine ungrouped numerical data to determine: The middlemost score in the data set (median)	• Samples and populations in data collection • Using simple questionnaires (yes/no type response and multiple choice questionnaires) • Using Stem and Leaf displays and grouping data into intervals to process the data correctly. • Draw a variety of graphs to display and interpret grouped and ungrouped data including: Histograms with given intervals and Pie charts • Deciding on the appropriate scale for a graph • Analyse data by answering questions related to: Central Tendencies – (mean, mode and median) and Dispersion – (determining the lowest and the highest values and then the range) • Report on data that must include: Drawing a conclusion on the data • Making predictions based on data • Choosing appropriate summary statistics (mean, median and mode)	• Pose questions relating to social, economic, and environmental issues in own environment • Select appropriate sources for the collection of data (including peers, family, newspapers, books, magazines) • Distinguish between samples and populations and suggest appropriate samples for investigation • Design and use simple questionnaires to answer questions: • Organize numerical data in different ways in order to summarize by determining: Measures of central tendency and Measures of dispersion, including extremes and Outliers • Organize data according to more than one criteria • Critically read and interpret data represented in a variety of ways • Critically compare two sets of data related to the same issue • Critically analyse data by answering questions related to: • Data collection methods • Summary of data • Sources of error and bias in the data • Summarize data in short paragraphs that include: • Drawing conclusions about the data • Making predictions based on the data • Making comparisons between two sets of data • Identifying sources of error and bias in the data • Choosing appropriate summary statistics for the data (mean, median, mode, range) • The role of extremes and outliers in the data

GLOSSARY OF TERMS

Term	Explanation / Diagram
Investigate	To collect data based on a question or several questions in order to draw a conclusion
Compare	To look at the data critically especially when collected from different sources and looking for common factors as well as differences
Questionnaire	A set of questions used in the investigation that delivered the data to be used in the data cycle
Population	The group that will be influenced by the outcome of the data collected
Sample	The group chosen to complete the questionnaire and to influence the data collected
Bias	When certain groups are excluded in the collection of data so that the results are manipulated to deliver a desired result
x axes and y axes	x axes –horizontal and y axes –vertical
Mode	This is the number that appears the most in the data set.
Mean	This is the average – this is the sum of the data divided by the number of data.
Median	This is the number in the middle, if data is arranged from lowest to highest / highest to lowest.
Range	This is the difference between the largest and the smallest numbers of the data.
Tally	This is an easy way to add up data when you are collecting information from your data source.  5 The 4 lines represent 4 items counted and the 5th is a cross through the 4.
Bar graph	A bar graph is a way of showing data that uses horizontal or vertical rectangular bars.
Double Bar Graph	A bar graph where more than one data set is being compared to another
Pie graph	A pie graph is a way of showing what fraction of data is represented by each category.
Pictographs **One-to-one pictograph** **Many-to-one pictograph**	A pictograph uses pictures or symbols to represent an amount of data. The key for a pictograph tells the number that each picture or symbol represents. Each picture of an animal represents one item Each picture represents several items
Stem and leaf display	A method to organise numerical data in order of place value. The "tens and greater" digit would represent the stems and the "ones" digit would be the leaves.
Histogram	A bar type graph used for grouped data as intervals would border on each other

SUMMARY OF KEY CONCEPTS

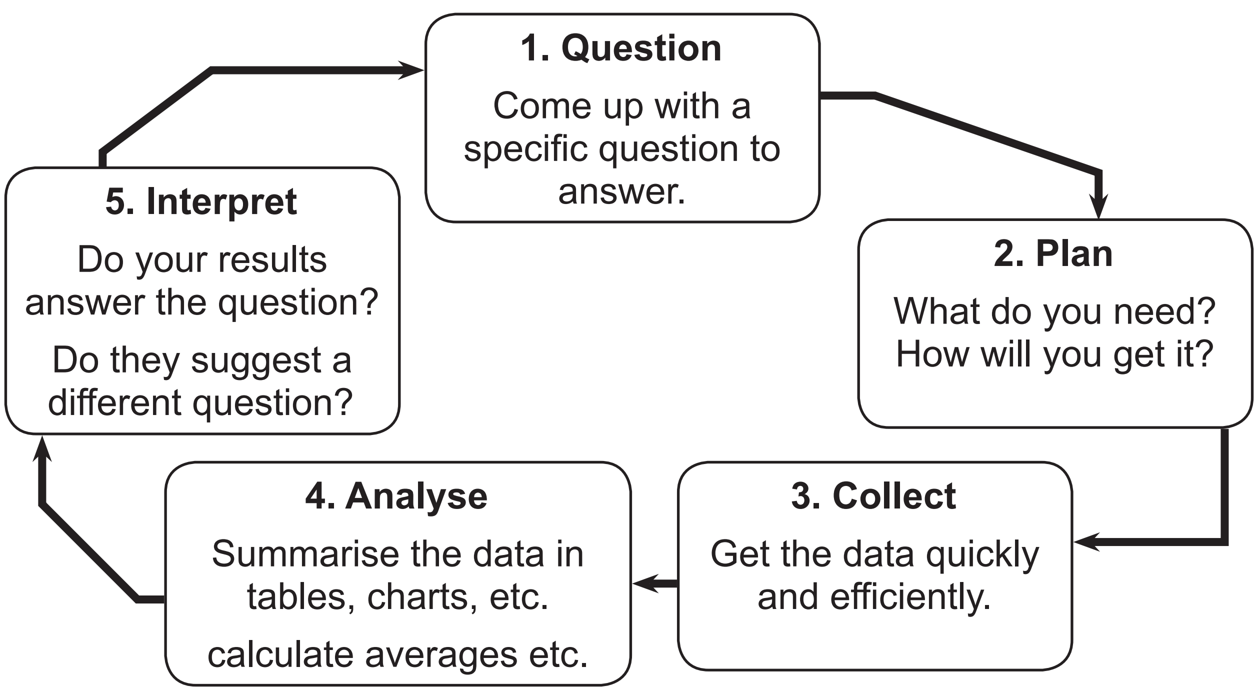
Data sets and contexts

1. Learners should be exposed to a large variety of contexts that deal with social and environmental issues.
2. Learners should work with given data sets represented various ways that must include large number ranges, percentages and decimal fractions.
3. Learners should get enough practise organising and summarising the data, analysing and interpreting the data and writing reports – in short paragraph form – on the data.

Complete a data cycle

1. The complete data cycle includes posing a question, collecting, organising, representing, analysing, interpreting data and reporting on the data.
2. Learners should complete one data cycle for the year.
3. Challenge learners to think about what types of questions will give the correct types of data to be represented in bar graphs, histograms and pie charts.

DATA HANDLING CYCLE



Collecting and ordering data

1. Data Collection

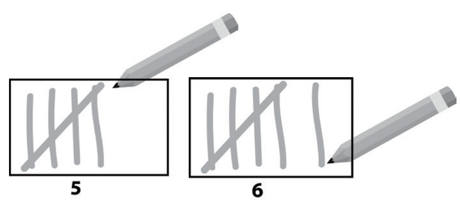
   Data can be collected in different ways. Learners must select the appropriate method for data collection. Sources could include peers, newspapers, family, books or magazines. The simplest way is observation.

   Example:
   You would like to know how many children are buying bread and milk from the local shop.
   So: stand at the door of the local shop and look at the number of children leaving with bread and milk in their packets.

2. Learners must pose questions that relate to social, economic or environmental issues in their own environment.
3. Learners must distinguish clearly between samples and populations.
4. Learners must be able to set yes/no questionnaires as well as multiple choice questionnaires at this grade level.
5. Data can be collected by doing surveys

   To do this:
   Survey people (through questionnaires, opinion polls, etc.)
   or things (like pollution levels in a river, or favourite colours).
6. Here are four steps to a successful survey:
   - Draw up the questions
   - Ask the questions
   - Tally the results
   - Present the results
7. Data can be recorded by using tally marks.
   This is how you record the items:

Interval	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	1	2	5	3	3	2	6	0	0	2



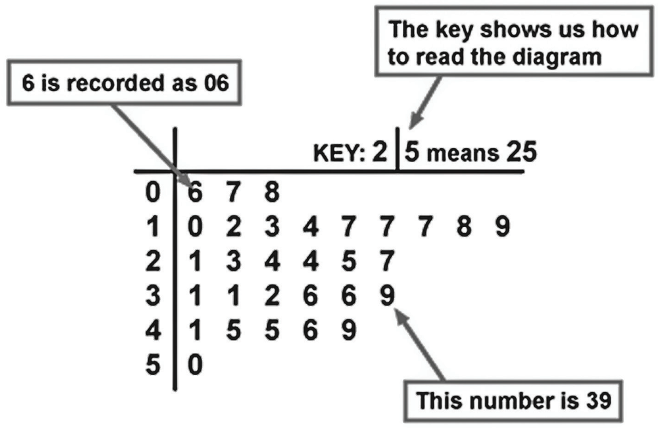
7,8 and 9…When you get to 10 then you mark the 6-9 off with a line through like you did for 5.

8. When doing a simple survey, tally each person's answers:

Our Favourite Sport

Sport	Tally	Number of Students
Hockey	𝍸 \	6
Soccer	𝍸	5
Basketball	\\\\	4
Gymnastics	\\	2

9. Data can also be organised using stem and leaf displays:



10. Grouping data into the correct intervals is also a new skill that must be mastered by learners. When a data set contains many different values we may often group data into sets to make it easier for data to be analysed.

11. Example:
The 24 learners of a local Sunday school have been collecting bottles for recycling. The list of values below shows the number of bottles each learner collected:

9; 10; 13; 23; 24; 26; 26; 27; 30; 31; 34; 40; 42; 49; 50; 53; 61; 64; 67; 67; 68; 69; 91; 94

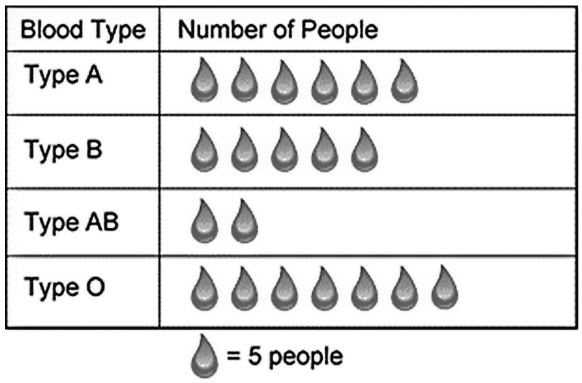
12. This data is best organised into class intervals (groups) as follows:

Interval	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	1	2	5	3	3	2	6	0	0	2

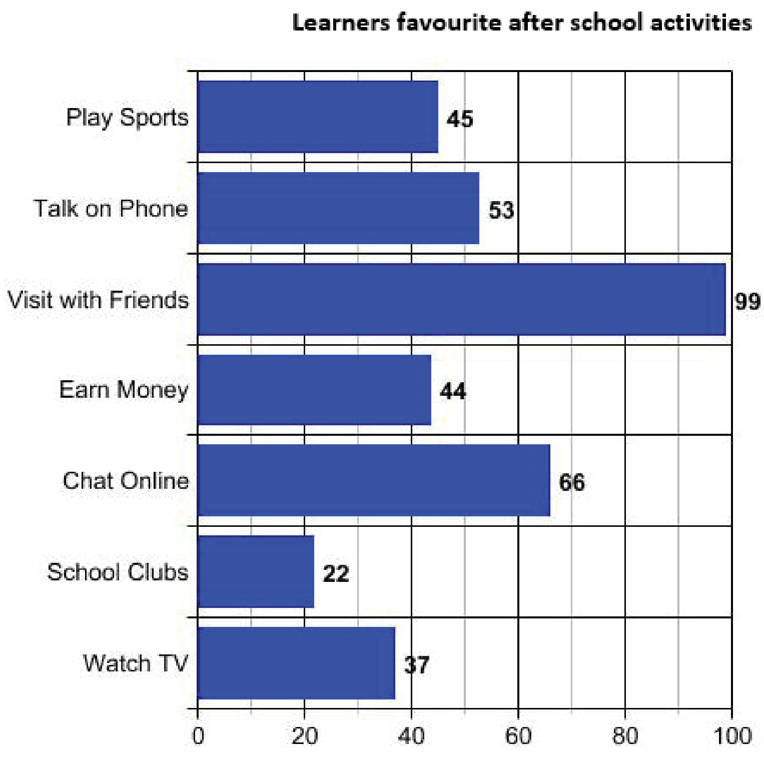
Types of graphs

1. Pictograph: many-to-one

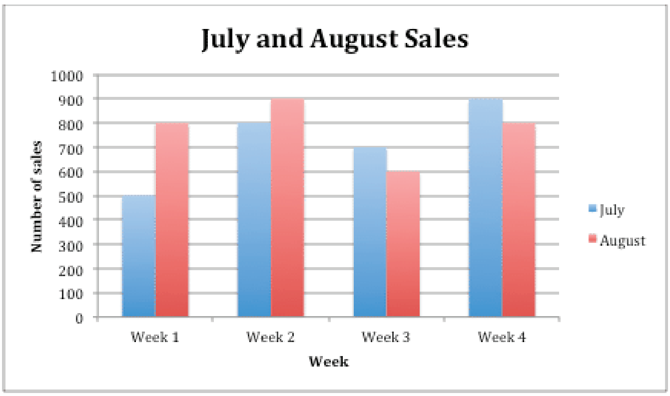
   This type of graph uses images to represent what is being compared.
   This is a many-to-one as one picture of a drop represents 5 people



2. Bar graph

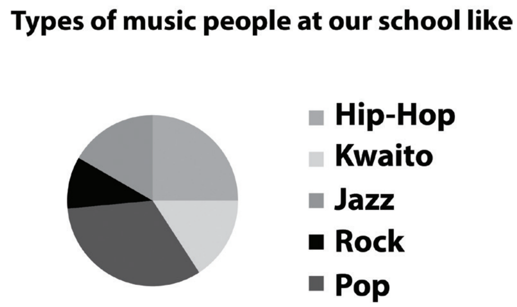


Double bar graph

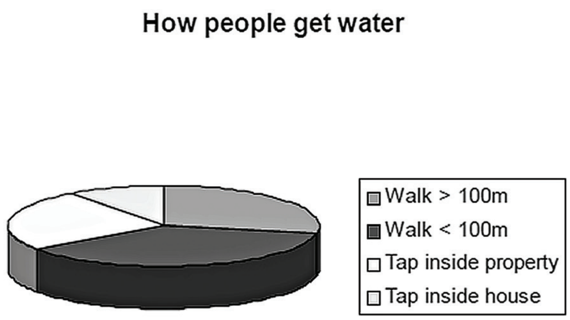


This bar graph represents a comparative of sets of data for July and August. The data is compared directly to determine a trend and this makes it easier for learners to analyse the data by comparison.

3. Pie graphs or pie charts



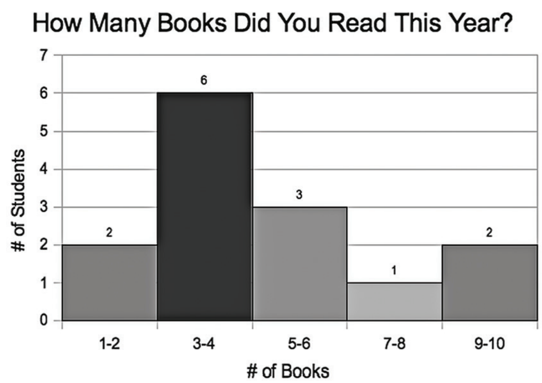
- Each slice is a fraction of the whole and is calculated as a fraction of the circle.
- Each slice represents what percentage or fraction of people like each type of music.
- Beware of using 3D pie charts when explaining this concept as learners must first master the skills in 2D pie charts before experimenting with these.



4. Learners may use any round object to draw the pie chart and do not need to use compass and protractor at this stage. Learners can divide the circle into half, quarters or eighths (if necessary). It is important that learners know that the representation within the pie chart must be proportional to the total area of the circle.

Teaching Tip:
Drawing, reading and interpreting pie charts is a useful context to re-visit the equivalence between fractions and percentages. It is also a good context that can be used to once again explain the determination of percentages as parts of a whole.

5. Histograms are used to represent grouped data shown in intervals on the horizontal axis of the graph.



Teaching Tip:
Point out the differences between bar graphs, compound bar graphs and histograms and the relevance of each type of graph and when it is suitable to use which one. A quick comparative exercise will be an excellent way to help learners see the differences.

Analysing graphs

1. Analyse graphs on environmental or socio-economic contexts by answering questions on graphs.
2. Both graphs and questions must be provided by the teacher or a textbook.
3. The questions that learners answer should be related to:
   - Data categories, including data intervals
   - Data sources and contexts
   - Central tendencies such as mean, median and mode
   - Scales on the graphs and how they influence the value of the data

Developing Critical analysis skills

1. Learners compare graphs on the same topic but where data has been collected from different groups of people, at different times, in different places or in different ways.
2. Here learners will be able to discuss the differences between the data with a clear awareness of bias related to the impact of data sources and the method used for data collection.
3. Learners should compare the same data represented in a variety of ways. Learners must discuss what information is shown and what is hidden away.
4. Learners should evaluate what form of representation is most suitable for the given data.
5. They should compare different ways of summarising the same data sets, developing an awareness of how data reporting can be manipulated. This only relates to how the data has been represented to change way the data is interpreted.
6. Learners should compare graphs where the scales have been manipulated to change the data's value represented.
7. Reports (short paragraphs) need to be written by learners on data that has been analysed.

Summary of Statistical Terms

1. Mode

This is the number that appears the most.
3, 3, 3, 6, 7, 7, 13
3 is the mode

There can be two modes in a set of data
There can also be no mode in a set of data

2. Mean

This is the average – this is the sum of the data divided by the number of data.
e.g. $3 + 3 + 3 + 6 + 7 + 7 + 13 = 42$
$42 \div 7 = 6$
6 is the mean

3. Median

This is the number in the middle, if data is arranged from lowest to highest.
e.g. 3, 3, 3, 6, 7, 7, 13
6 is the middle number

4. Range

This is the difference between the largest and the smallest numbers of the data.

e.g. 3, 3, 3, 6, 7, 7, 13

$13 - 3 = 10$

Teaching Tip:

A good understanding of each component in this section is important for the later stages of mathematics especially since this section has been included in the curriculum. Learners entering tertiary studies will need to have a knowledge of the content covered in Statistics

TOPIC 7: PROBABILITY

INTRODUCTION

- This unit runs for 4,5 hours.
- It is part of the content area, 'Data Handling' and counts for 10 % in the final exam.
- The unit is an extension of the work done in previous grades and includes probability by definition.
- This term learners are expected to develop an understanding of experimental probability and to start developing an understanding of theoretical probability.
- Practice as much as possible – if the textbook used does not have enough exercises in, re-do some, find some more or make up a worksheet of your own.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 6	GRADE 7	GRADE 8 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Perform single repeated events and list the possible outcomes for experiments such as rolling a die, flipping a coin and spinning a simple spinner. • Count and compare the frequency of actual outcomes for a series of trials up to 50 trials.	• Perform simple experiments where the possible outcomes are equally likely. • List the possible outcomes based on the conditions of an activity • Determine the probability of each possible outcome, using the definition of probability.	• Relative frequency experiments and theoretical probability. • The application of theoretical probability to determine chance. • Venn diagrams and proving chance by calculations.

GLOSSARY OF TERMS

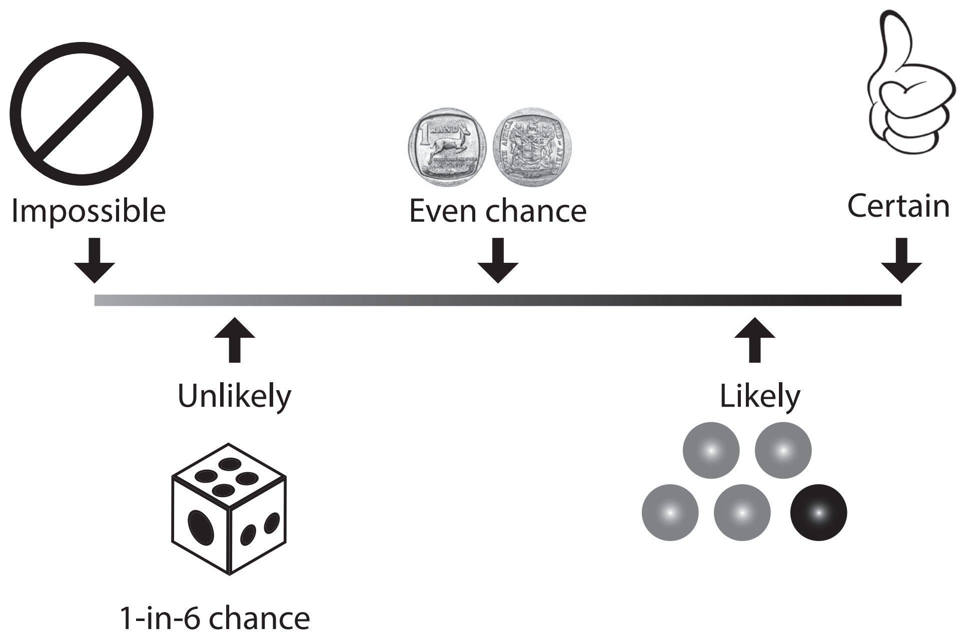
Term	Explanation / Diagram
Experiment	Something we do to find out what will happen
Trial	The activity that is used to determine what will happen
Outcome	The result of the trial
Frequency	How often the outcome occurs
Event	Composed of the outcomes of an experiment
Probability	The likelihood of something happening. Usually expressed as a fraction (but could also be expressed as a decimal or a percentage). A probability answer is always in the range $0 \leq x \leq 1$. In other words, the answer can only be 0; 1; or a common fraction
Definition of Probability	$\dfrac{\textit{The number of outcomes in the event that satisfy the experiment}}{\textit{the number of all possible outcomes}}$

SUMMARY OF KEY CONCEPTS

An important concept of probability that needs to be understood from the beginning is that no answer to a probability question can ever be less than zero or bigger than one.

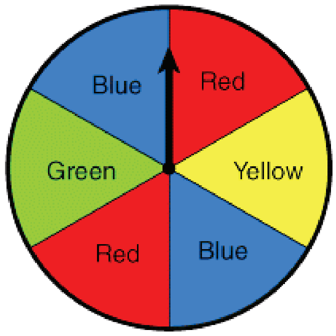
Every answer will always lie somewhere from zero to 1. Therefore, most answers are fractions except those that are actually zero or one.
Remember that fractions can be written in more than one way.
For example, $\frac{1}{2} = 0,5 = 50\%$

If there is an equally likely chance of an event happening (also known as a 50/50 chance), any one of the above fractions would represent the probability of such an event occurring.
(Remember that 50% means 50 divided by 100 and is therefore not a number bigger than one although it may at first appear to be so)



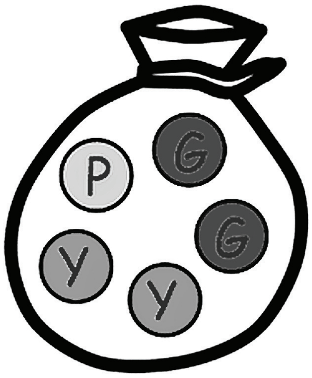
1. Learners should be able to determine probability by trial using a variety of methods. These methods would be expanded to include: coloured buttons in a bag or packs of cards.

EXAMPLE:
Spinning a spinner



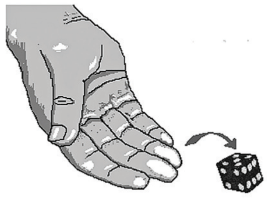
The spinner could land on red or blue twice but on yellow or green only once each.

Bags of buttons or marbles



Flipping a coin

Rolling a die



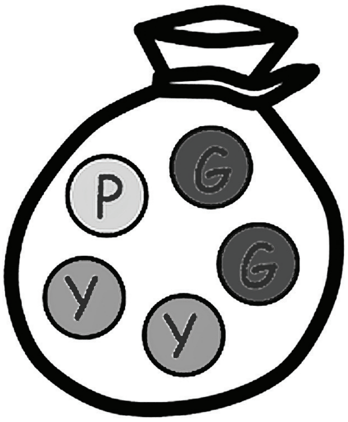
Packs of cards

Calculating probability using the definition

1. Learners must use the definition of probability to determine the outcomes of trials and experiments.

$$\frac{\textit{The number of outcomes in the event that satisfy the experiment}}{\textit{the number of all possible outcomes}}$$

Example:



The probability of picking a pink marble from the bag is:

$$\frac{\textit{The number of outcomes in the event that satisfy the experiment}}{\textit{the number of all possible outcomes}}$$

$$= \frac{1 \textit{ pink marble}}{6 \textit{ marbles in the bag}}$$

Note: Learners will use this skill later when they are working with theoretical probability and Venn diagrams in the senior and FET phases. Learners will use this to make predictions based on the definition of probability so starting the development of this skill now will assist learners later on.

Where possible, allow learners to spend time playing and trying out experiments and trials for themselves.