‘There is geometry in the humming of the strings. There is music in the spacing of the spheres.’

–**Pythagoras**

MATHEMATICS

LESSON PLAN

GRADE 11 TERM 4

MESSAGE FROM NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE). We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

WHAT IS NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

WHAT ARE THE LEARNING PROGRAMMES?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Universalisation Programme and in its Provincialisation Programme.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

CONTENTS

PROGRAMME ORIENTATION

Welcome!

The NECT FET Mathematics Learning Programme is designed to support teachers by providing:

- Lesson Plans
- Trackers
- Resource Packs
- Assessments and Memoranda
- Posters.

This Mathematics Learning Programme provides most of the planning required to teach FET Mathematics. However, it is important to remember that although the planning has been done for you, preparation is key to successful teaching. Set aside adequate time to properly prepare to teach each topic.

Also remember that the most important part of preparation is ensuring that you develop your own deep conceptual understanding of the topic. Do this by:

- working through the lesson plans for the topic
- watching the recommended video clips at the end of the topic
- completing all the worked examples in the lesson plans
- completing all activities and exercises in the textbook.

If, after this, a concept is still not clear to you, read through the section in the textbook or related teacher's guide, or ask a colleague for assistance. You may also wish to search for additional teaching videos and materials online.

Orientate yourself to this Learning Programme by looking at each component, and by taking note of the points that follow.

TERM 4 TEACHING PROGRAMME

1. In line with CAPS, the following teaching programme has been planned for FET Mathematics for Term 3:

Grade 10		Grade 11		Grade 12	
Topic	**No. of weeks**	**Topic**	**No. of weeks**	**Topic**	**No. of weeks**
Probability	2	Statistics	3	Revision	3
Revision	4	Revision	3		

2. Term 4 lesson plans and revision plan are provided for six weeks for Grades 10 and 11.
3. Term 4 revision plans are provided for three weeks for Grade 12
4. Each week includes 4,5 hours of teaching time, as per CAPS.
5. You may need to adjust the lesson breakdown to fit in with your school's timetable.

LESSON PLAN STRUCTURE

The Lesson Plan for each term is divided into topics. Each topic is presented in exactly the same way:

TOPIC OVERVIEW

1. Each topic begins with a brief **Topic Overview**. The topic overview locates the topic within the term, and gives a clear idea of the time that should be spent on the topic. It also indicates the percentage value of this topic in the final examination, and gives an overview of the important skills and content that will be covered.

2. The **Lesson Breakdown Table** is essentially the teaching plan for the topic. This table lists the title of each lesson in the topic, as well as a suggested time allocation.
   For example:

	Lesson title	Suggested time (hours)
1	Revision	2,5
2	Venn diagrams	2,5
3	Inclusive and mutually exclusive events; Complementary and Exhaustive events	1,5
4	Revision and Consolidation	1,5

3. The **Sequential Table** shows the prior knowledge required for this topic, the current knowledge and skills to be covered, and how this topic will be built on in future years.
   - Use this table to think about the topic conceptually:
     - Looking back, what conceptual understanding should learners have already mastered?
     - Looking forward, what further conceptual understanding must you develop in learners, in order for them to move on successfully?
   - If learners are not equipped with the knowledge and skills required for you to continue teaching, try to ensure that they have some understanding of the key concepts before moving on.
   - In some topics, a revision lesson has been provided.
4. The **NCS Diagnostic Reports**. This section is potentially very useful. It lists common problems and misconceptions that are evident in learners' NSC examination scripts. The Lesson Plans aim to address these problem areas, but it is also a good idea for you to keep these in mind as you teach a topic.
5. The **Assessment of the Topic** section outlines the formal assessment requirements as prescribed by CAPS for Term 4.

Grade	Assessment requirements for Term 4 (as prescribed in CAPS)
10	Test, Examination Paper I and Paper II
11	Test, Examination Paper I and Paper II
12	Examination Paper I and Paper II

6. The glossary of **Mathematical Vocabulary** provides an explanation of each word or phrase relevant to the topic. In some cases, an explanatory sketch is also provided. It is a good idea to display these words and their definitions or sketches somewhere in the classroom for the duration of the topic. It is also a good idea to encourage learners to copy down this table in their free time, or alternately, to photocopy the Mathematical Vocabulary for learners at the start of the topic. You should explicitly teach the words and their meanings as and when you encounter these words in the topic.

INDIVIDUAL LESSONS

1.. Following the **Topic Overview**, you will find the **Individual Lessons.** Each lesson is structured in exactly the same way. The routine within the individual lessons helps to improve time on task, and therefore, curriculum coverage.

2. In addition to the lesson title and time allocation, each lesson plan includes the following:

A. ***Policy and Outcomes***. This provides the CAPS reference, and an overview of the objectives that will be covered in the lesson.

B. ***Classroom Management***. This provides guidance and support as you plan and prepare for the lesson.

- Make sure that you are ready to begin your lesson, have all your resources ready (including resources from the Resource Pack), have notes written up on the chalk-board, and are fully prepared to begin.
- Classroom management also suggests that you plan which textbook activities and exercises will be done at which point in the lesson, and that you work through all exercises prior to the lesson.
- In some cases, classroom management will also require you to photocopy an item for learners prior to the lesson, or to ensure that you have manipulatives such as boxes and tins available.

***The Learner Practice Table*.** This lists the relevant practice exercises that are available in each of the approved textbooks.

- It is important to note that the textbooks deal with topics in different ways, and therefore provide a range of learner activities and exercises. Because of this, you will need to plan when you will get learners to do the textbook activities and exercises.
- If you feel that the textbook used by your learners does not provide sufficient practice activities and exercises, you may need to consult other textbooks or references, including on0line references.
- The *Siyavula* Open Source Mathematics textbooks are offered to anyone wishing to learn mathematics and can be accessed on the following website: https://www.everythingmaths.co.za/read

C. ***Conceptual Development***:

This section provides support for the actual teaching stages of the lesson.

Introduction: This gives a brief overview of the lesson and how to approach it. Wherever possible, make links to prior knowledge and to everyday contexts.

Direct Instruction: Direct instruction forms the bulk of the lesson. This section describes the teaching steps that should be followed to ensure that learners develop conceptual understanding. It is important to note the following:

- Grey blocks talk directly to the teacher. These blocks include teaching tips or suggestions.
- Teaching is often done by working through an example on the chalkboard. These worked examples are always presented in a table. This table may include grey cells that are teaching notes. The teaching notes help the teacher to explain and demon-strate the working process to learners.

- As you work through the direct instruction section, and as you complete worked examples on the chalkboard, ensure that learners copy down:
  - formulae, reference notes or explanations
  - the worked examples, together with the learner's own annotations.
- These notes then become a reference for learners when completing examples on their own, or when preparing for examinations.
- At relevant points during the lesson, ensure that learners do some of the Learner Practice activities as outlined at the beginning of each lesson plan. Also, give learners additional practice exercises and questions from past papers as homework. Ensure that learners are fully aware of your expectations in this respect.

D. ***Additional Activities / Reading****.* This section provides you with web links related to the topic. Get into the habit of visiting these links as part of your lesson preparation. As teacher, it is always a good idea to be more informed than your learners. If possible, organise for learners to view video clips that you find particularly useful.

THE REVISION PROGRAMME

The teaching programme for FET mathematics Term 4 differs from the teaching programmes for Terms 1-3. There is only one topic with new content in Term 4 for Grades 10 and 11; and no new content in Term 4 for Grade 12. Most of the contact time in Term 4 is allocated to consolidation, revision and preparation for the end of year examinations. The Revision Programme for each grade are designed to support you and the learners so as to ensure that revision time is effectively and productively used.

THE STRUCTURE OF THE REVISION PROGRAMME

- Summary notes for the topics assessed in Paper I and Paper II. These notes are provided in the Resource Pack. If possible, the summary notes should be photocopied for learners. Alternatively, you could provide learners with an electronic copy of the summary notes; or learners can copy down the summary notes. Encourage learners to add their own notes to the summary notes you have given them.
- Fully worked past paper
- Past papers, exemplars and memoranda. The past papers, exemplars and memoranda are provided in the Resource Pack. If possible, the past papers, exemplars and memoranda should be photocopied for learners. Alternatively, you could provide learners with an electronic copy of the examinations, exemplars and memoranda; or learners can share copies. The links to these resources are provided in the Lesson Plan.

Working through past papers and exemplars has been shown to be an excellent learner-centred approach to revision. For this reason, we urge you to do everything possible to ensure that learners have access to these materials.

TRACKER

1. A Tracker is provided for Grades 10 and 11 for Term 4. The Trackers are CAPS compliant in terms of content and time.
2. You can use the Tracker to document your progress. This helps you to monitor your pacing and curriculum coverage. If you fall behind, make a plan to catch up.
3. Fill in the Tracker on a daily or weekly basis.
4. At the end of each week, try to reflect on your teaching progress. This can be done with the HoD, with a subject head, with a colleague, or on your own. Make meaningful notes about what went well and what didn't. Use the reflection section to reflect on your teaching, the learners' learning and to note anything you would do differently next time.
   These notes can become an important part of your preparation in the following year.

RESOURCE PACK, ASSESSMENT AND POSTERS

1. A Resource Pack with printable resources has been provided for each term.
2. These resources are referenced in the lesson plans, in the Classroom Management section.
3. Two posters have been provided as part of the FET Mathematics Learning Programme for Term 4.
4. Ensure that the posters are displayed in the classroom.
5. Try to ensure that the posters are durable and long-lasting by laminating it, or by covering it in contract adhesive.
6. Note that you will only be given these resources once. It is important for you to manage and store these resources properly. You can do this by
   - Writing your school's name on all resources
   - Sticking resource pages onto cardboard or paper
   - Laminating all resources, or covering them in contact paper
   - Filing the resource papers in plastic sleeves once you have completed a topic.

7. Add other resources to your resource file as you go along.

8. Note that these resources remain the property of the school to which they were issued.

ASSESSMENT AND MEMORANDUM

In the Resource Pack you are provided with assessment exemplars and memoranda as per CAPS requirements for the term. For Term 4, the Resource Pack contains one test and memorandum for Grades 10 and 11. In addition, past papers, exemplars and memoranda are provided for Grades 10, 11 and 12.

CONCLUSION

Teacher support and development is a complex process. For successful Mathematics teachers, certain aspects of this Learning Programme may strengthen your teaching approach. For emerging Mathematics teachers, we hope that this Learning Programme offers you meaningful support as you develop improved structure and routine in your classroom, develop deeper conceptual understanding in your learners and increase curriculum coverage.

Term 4, Topic 1: Topic Overview

STATISTICS

A. TOPIC OVERVIEW

A

- This is the only topic in Term 4.
- This topic runs for three weeks (13,5 hours).
- It is presented over six lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 13,5 hours). For example, one lesson in this topic could take three school lessons. Plan according to your school's timetable.
- Statistics counts 13% of the final Paper 2 examination.
- This is a section of work in which learners can score high marks. Make a concerted effort to ensure learners understand this topic. This topic should not be relegated to a rushed job at the end of the year. (Diagnostic report).
- At least half of the marks in a Grade 12 exam are made up of concepts from Grade 10 and Grade 11.

Breakdown of topic into 6 lessons:

	Lesson title	**Suggested time (hours)**		**Lesson title**	**Suggested time (hours)**
1	Revision of Grade 10 statistics	2,5	4	Variance and Standard deviation	2,5
2	Histograms and frequency polygons	1,5	5	Symmetric and skewed data; Identification of outliers	2,5
3	Cumulative frequency curves (Ogives)	2,5	6	Revision and consolidation	2

B SEQUENTIAL TABLE

GRADE 10 and Senior phase	GRADE 11	GRADE 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Measures of central tendency in grouped and ungrouped data • Estimated mean of grouped data • Modal interval and interval in which median lies • Five number summary • Box and whisker diagrams • Measures of dispersion to include range, percentiles, quartiles, interquartile range and semi-interquartile range.	• Histograms • Frequency polygons • Ogives • Variance and standard deviation of ungrouped data • Symmetric and skewed data • Identification of outliers.	Use: • statistical summaries • scatterplots • regression (a least squares regression line) • correlation to analyse and make meaningful comments on the context associated with given bivariate data, including interpolation, extrapolation and discussions on skewness.

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are several issues pertaining to Data Handling.

These include:

- inability to complete a cumulative frequency table
- poor understanding of a frequency column
- inability to calculate the standard deviation correctly

It is important, as the teacher, that you keep these issues in mind when teaching this section.

While teaching statistics, ensure that learners understand the terms required in this section. For example, grouped and ungrouped data. Correct statistical vocabulary and terminology must always be used.

Learners should be exposed to real life scenarios and answer many different types of questions (particularly those of an interpretive nature) to improve their performance. Exposure to these types of questions cannot be over-emphasised. It should form an integral part of the teaching and learning of this topic.

ASSESSMENT OF THE TOPIC

D

- CAPS formal assessment requirements for Term 4:
  - Test
  - Examination (Paper 1 & Paper 2)
- A test, with memorandum, is provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53).
- The questions usually take the form of being given a set of data, finding measures of central tendency and dispersion and needing to find trends.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

MATHEMATICAL VOCABULARY

E

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	**Explanation**
data	Facts or information collected from people or objects. Data is plural for datum
population	The entire group of people or objects that data is being collected from
sample	A smaller part of the population if the population is too large
random	How to choose a smaller sample of the population to attempt to not be biased
questionnaire	A set of printed questions with a choice of answers used in the data collection process
survey	The collecting of data from a group of people
discrete data	Data that can only take certain values. For example, the number of learners in a class (there can't be half a learner)

continuous data	Data that can take on any value within a certain range. For example, the heights of a group of learners (heights could be measured in decimals)
tally	A way of keeping count by drawing marks Every fifth mark is drawn across the previous four (to form a gate-like diagram) so you can easily see groups of five
measures of central tendency	A single value that describes the way in which a group of data cluster around a central value There are three measures of central tendency: the mean, the median, and the mode
mean	The average of a set of numbers. Calculated by adding all the values then dividing by how many numbers there are
median	The middle number in a sorted list of numbers To find the median, place all numbers in order from smallest to biggest and find the middle number
mode	The number that appears the most often in a set of data. There can be two modes. There could also be no mode in a set of data
modal class	The class with the highest frequency from a set of grouped data. in other words, the interval with the most “members”
measures of dispersion	Measures of dispersion like the range, percentiles and quartiles tell you about the spread of scores in a data set Like central tendency, measures of dispersion help you summarise a set of data with one or just a few numbers
range	The difference between the highest value and lowest value in a set of data
percentiles	Each of the 100 equal groups into which a population can be divided according to the distribution of values of a variable The value below which a percentage of data falls
quartiles	Each of four equal groups into which a population can be divided according to the distribution of values of a variable The values that divide a list of numbers into quarters

interquartile range (IQR)	The interquartile range is a measure of variability, based on dividing a data set into quartiles Quartiles divide a rank-ordered data set into four equal parts The values that divide each part are called the first, second, and third quartiles; and they are denoted by Q1, Q2, and Q3 respectively
histogram	A graph representing data that is grouped into ranges and each bar represents data that follows on from the previous bar. Example, one bar could represent how many learners got a mark from 40-49 and the bar immediately next to it would represent 50-59.
scatter plots	A graph in which the values of two variables are plotted along two axes. The pattern of the resulting points reveals whether there is any correlation between the two sets of values
line of best fit	A straight line drawn through the centre of a group of data points plotted on a scatter plot. Scatter plots depict the results of gathering data on two variables
outliers	Values that are significantly higher or lower than all the other values in the data set. Outliers are also called extremes. Outliers can affect the mean of the data and are sometimes excluded when calculations are done
ungrouped data	Ungrouped data has not been classified or has not been subdivided in the form of groups. Ungrouped data is raw data Ungrouped data is in the form of a list of numbers
grouped data	Data that has been ordered and sorted into groups called classes
estimated mean	An estimate of the mean can be determined for grouped data. Unlike listed data, the individual values for grouped data are not available, and it is not possible to calculate their sum. To calculate the mean of grouped data, first determine the midpoint of each interval, or class. The midpoints must then be multiplied by the frequencies of the corresponding classes. The sum of the products divided by the total number of values will be the value of the mean
five number summary	Lowest value, lower quartile, median, upper quartile and highest value from a set of data The five numbers are used to draw a box-and-whisker plot

box-and-whisker plot	A simple way of representing statistical data on a plot in which a rectangle is drawn to represent the second and third quartiles, usually with a vertical line inside to indicate the median value. The lower and upper quartiles are shown as vertical lines either side of the rectangle. The lowest value and highest value in the data set are represented at each end
frequency table	A table that lists a set of scores and their frequency. Often used with tallies. Summarises the totals and shows how often something has occurred
frequency polygon	Frequency polygons are a graphical device for understanding the shapes of distributions. Frequency polygons serve the same purpose as histograms but are especially helpful for comparing sets of data. Frequency polygons are an effective way of displaying cumulative frequency distributions
ogive	A cumulative frequency graph Ogives can be used to determine how many data values lie above or below a certain value in a data set
variance	A measure of the spread of a data set Variance is the average of the squared differences from the mean
standard deviation	A quantity expressing by how much the members of a group differ from the mean value for the group Standard deviation is the square root of the variance

TERM 4, TOPIC 1, LESSON 1

REVISION

Suggested lesson duration: 2,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	39

Lesson Objectives

By the end of the lesson, learners will have revised:

- measures of central tendency in grouped and ungrouped data
- estimated mean of grouped data
- modal interval and interval in which median lies
- five number summary
- box-and-whisker diagrams
- measures of dispersion.

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 11 textbooks. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		VIA AFRIKA		CLEVER		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1	310			Qu's	300			11.1	444

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. In general, the results for Data handling in the final Grade 12 examinations are good.
2. This is good news – learners already have a basis to work from to further their understanding and become excellent at this section.
3. Take the time to revise concepts covered in previous grades.

DIRECT INSTRUCTION

1. Start the lesson by saying: *Tell me what you can remember about data handling.* Allow learners to name concepts they remember, and then go through each concept discussing what it means. Learners should take notes.
2. The list of concepts and their explanations should come from the vocabulary list. Ensure learners understand all the vocabulary required for this section. The focus should be on the definitions as well as an understanding.
   Note: Do not discuss the last four concepts with learners as they are new to Grade 11. (Frequency tables, frequency polygons, ogive, variance and standard deviation)
3. This part of the lesson should take at least 45 minutes. Encourage learners to contribute to the discussion.

   > This rest of the lesson is made up of two fully worked examples from past Grade 10 papers covering all the concepts in this topic. As you work through these examples with the learners, discuss as many concepts as possible.
   > For example, use the words measures of central tendency, measures of dispersion, mean, median, mode (and modal class), range, quartiles, percentiles, estimated mean, five-number summary and box-and-whisker diagrams.

   Work through the two fully worked examples with learners. Learners should write them in full in their exercise books.

Example 1:	Teaching notes:
The data below shows the number of laptops sold by 15 sales agents during the last financial year. 43 48 62 52 46 90 58 37 48 73 84 68 54 34 78 a) Determine the median number of laptops sold.	Remind learners that to find the median, the data needs to be ordered.
b) Calculate the range of data.	Largest value – smallest value.
c) Calculate the interquartile range.	Find the upper quartile and lower quartile and subtract the lower quartile from the upper quartile.
d) Draw a box-and-whisker diagram for the data above. NSC NOV 2017	Use the five-number summary and remember to make sure the scale is accurate.

Solution:

Rearrange the data:

34 37 43 46 48 48 52 54 58 62 68 73 78 84 90

a) Median

$\frac{1}{2}(n+1)$

$= \frac{1}{2}(15+1)$

$= \frac{1}{2}(16)$

$= 8$

The median is in the 8th position. ∴ the median is 54

Note: Learners can count to find the median if they prefer.

b) 90 – 34 = 56

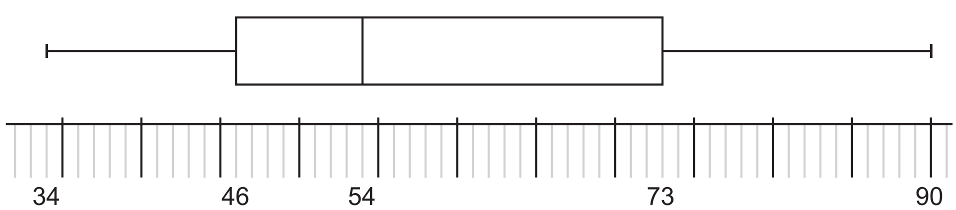
c) $Q_3 - Q_1$

$= 73 - 46$

$= 27$

d) Five-number summary:

34 46 54 73 90 (90 – 34 = 56. Suggested scale: 1cm : 5 units)



Example 2:	Teaching notes:
The table below shows information about the number of hours 120 learners spent on their cell phones in the last week.	
a) Identify the modal class for the data.	Find the class that has the most number of values
b) Estimate the mean number of hours that these learners spent on their cell phones in the last week. NSC NOV 2015	Find the midpoint of the class intervals and multiply by the frequency. Find the total of the products and divide by the number in the data set. Remind learners why they are doing this.

Number of hours	Frequency
$0 < h \leq 2$	10
$2 < h \leq 4$	15
$4 < h \leq 6$	30
$6 < h \leq 8$	35
$8 < h \leq 10$	25
$10 < h \leq 12$	5

Solution:

a) $6 < h \leq 8$

b)

Number of hours	Frequency	Midpoint of class intervals	Midpoint x frequency
$0 < h \leq 2$	10	1	10
$2 < h \leq 4$	15	3	45
$4 < h \leq 6$	30	5	150
$6 < h \leq 8$	35	7	245
$8 < h \leq 10$	25	9	225
$10 < h \leq 12$	5	11	55

Estimated mean

$$\frac{10 + 45 + 150 + 245 + 225 + 55}{120} = \frac{730}{120} = 6{,}08$$

$\therefore$ 6,08 hours

4. Ask directed questions so that you can ascertain learners' level of understanding.
   Ask learners if they have any questions.

5. Give learners an exercise to complete which they may need to finish at home. If this is the case, make sure you mark it in the next lesson before starting the new work.

6. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=KwpcKCX51ro

https://www.youtube.com/watch?v=kJrhyb6aG3A

(Estimated mean)

TERM 4, TOPIC 1, LESSON 2

HISTOGRAMS AND FREQUENCY POLYGONS

Suggested lesson duration: 1,5 hours

A

POLICY AND OUTCOMES

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners should be able to: • draw a frequency polygon • interpret a frequency polygon.	

B

CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw the two histograms (point 1).
5. The table below provides references to this topic in Grade 11 textbooks. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		VIA AFRIKA		CLEVER		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
		1 (1.1-1.3)	229	1 2	303 206	13.1	436	11.2	450

CONCEPTUAL DEVELOPMENT

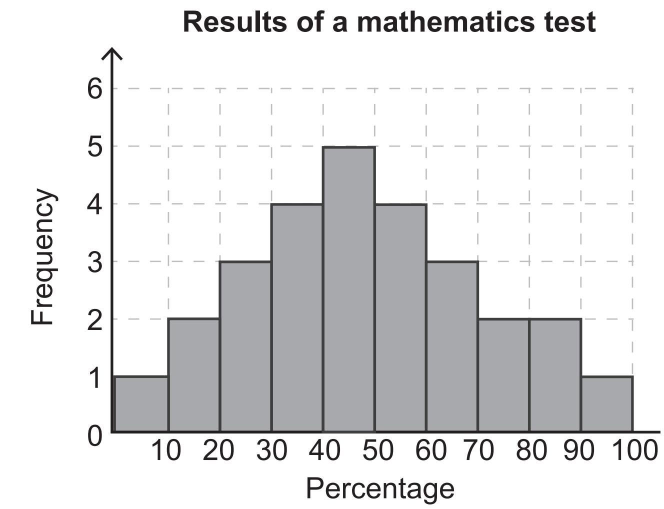
C

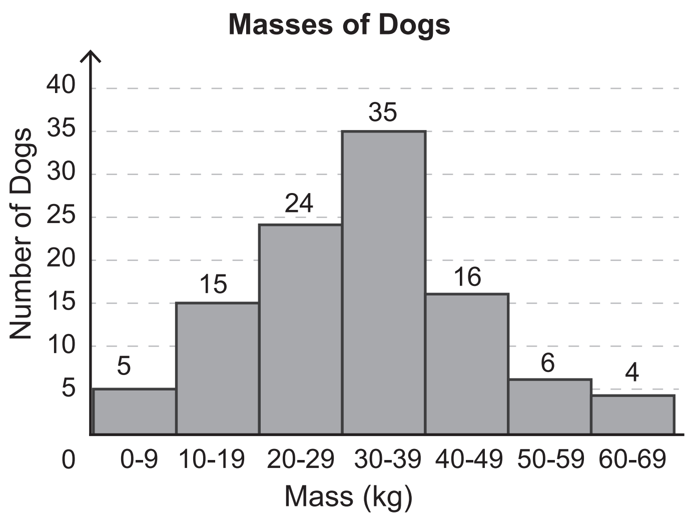
INTRODUCTION

1. Learners have encountered histograms in previous years. However, linking histograms to a frequency polygon is a new concept.

DIRECT INSTRUCTION

1. Start the lesson by asking learners to look at the following two histograms and to copy them into their books. Say: *As you are copying the histograms, think about what they represent as we are going to discuss them shortly.*





2. Ask: *What information can you provide about the first histogram?*
   Ensure the following points are mentioned. If any are not, ask directed questions to encourage learners to look for the information themselves.
   - We can see how many learners got from 0 – 10, 10 – 20 and 20 – 30 etc.
   - For example, one learner got less than 10 and there were three learners who each got between 20 and 30, as well as between 60 and 70.
   - We can find the total number of learners in the data set by adding all the class interval frequencies together (1+2+3+4+5+4+3+2+2+1=27).

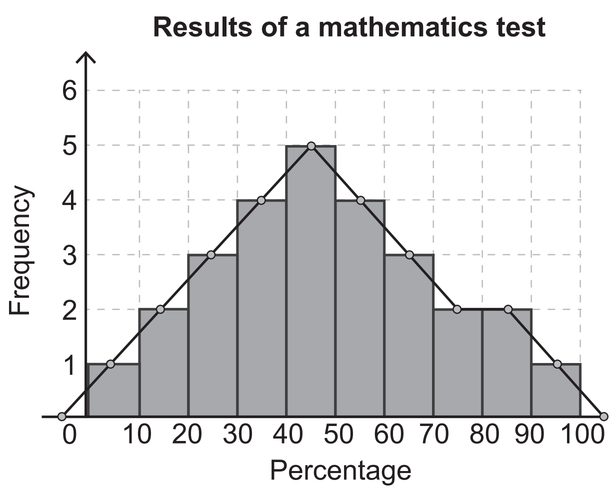
3. Use the above points to discuss the second histogram.
   - Five dogs had a mass between zero and 9kg, 15 dogs had a mass between 10kg and 19kg, four dogs had a mass between 60kg and 69kg etc.
   - 105 dogs in total were used for this set of data.

4. Say: *We are going to use these same histograms to discuss a new concept called a frequency polygon.*

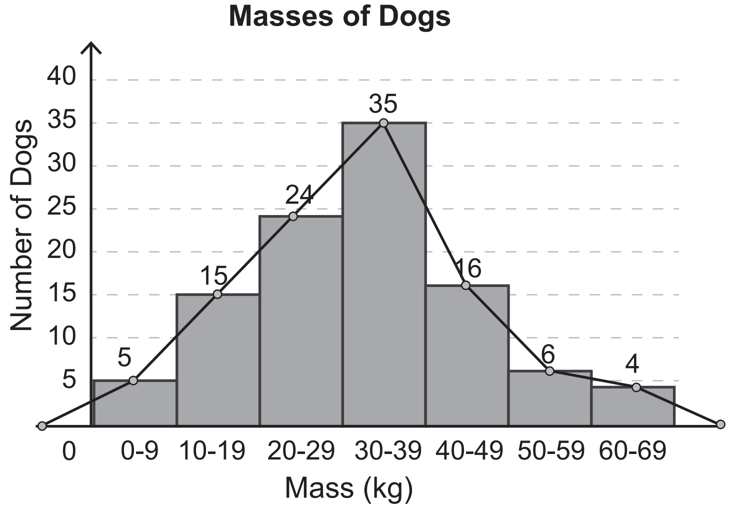
5. Tell learners that frequency polygons are a graphical representation which helps us understand the shapes of distributions. Frequency polygons serve the same purpose as histograms but are especially helpful for comparing sets of data.

6. Use the first histogram. Show learners how to draw a frequency polygon. Learners should follow the same steps to draw the histogram in their exercise books.

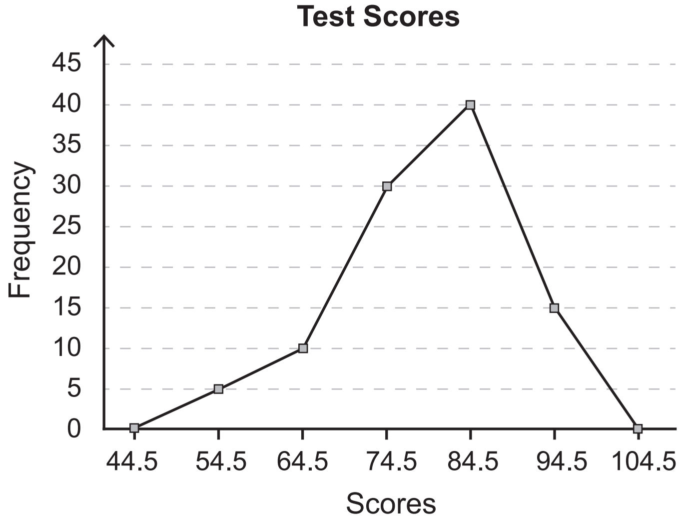
7. Steps to follow to draw a histogram:
   - Plot a point (at the top) in the centre of each bar.
   - Use a ruler. Join all the points.
   - Remind learners that we are drawing a polygon – a closed figure with straight sides – this means we need to close the shape.
   - Join the first point to an 'imaginary' point in the centre of the previous bar (which may also be imaginary and in this example is to the left of the vertical axis).
   - Join the last point to an 'imaginary' point in the centre of the following bar (which may also be imaginary and in this example can be the end of the horizontal axis).



8. Once the first frequency polygon has been completed, ask learners to work on their own to draw the second histogram. The histogram should look like this:



9. Tell learners that sometimes they may see the frequency polygon without the histogram. Draw this histogram on the board to demonstrate:



10. Discuss this frequency polygon with learners:
    - This frequency polygon represents test results.
    - Five learners got from 50 to 59 (remember the number represented is the centre of the bar and therefore class interval).
    - In total there were 100 learners.
    - No learner got less than 50%.
    - 15 learners got from 90% to 100%.
    - The modal class was 80% to 89% (there were 40 learners who achieved these percentages).

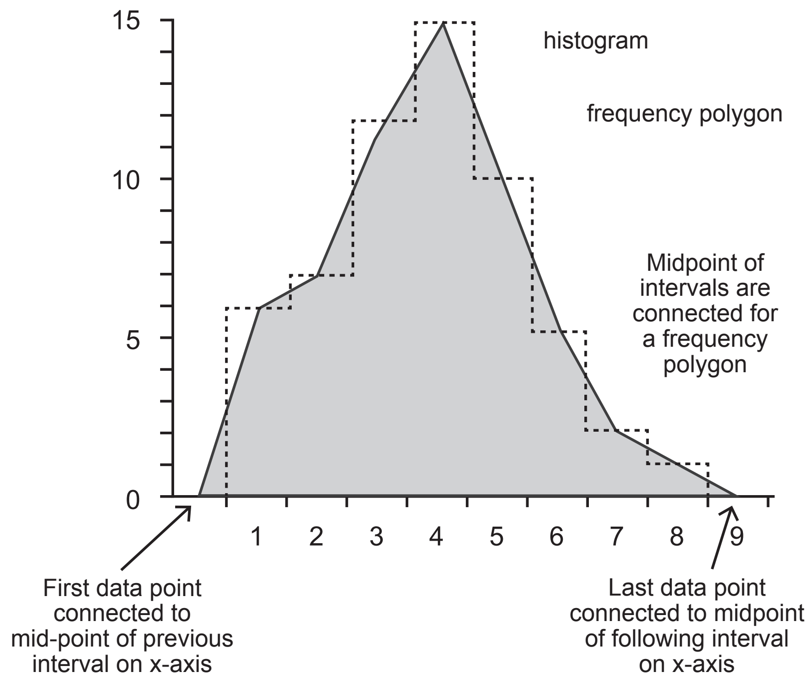
11. Point out that it is possible to link a question about estimated mean to frequency polygons. Find the estimated mean of this data now. Ask learners to write the table in their books.

Percentage	Frequency	Midpoint of class intervals	Midpoint x frequency
50-59	5	54,5	272,5
60-69	10	64,5	645
70-79	30	74,5	2235
80-89	40	84,5	3380
90-100	15	94,5	1417,5

Estimated mean

$$\frac{272,5 + 645 + 2235 + 3330 + 1417,5}{120} = \frac{7950}{100} = 79,5\%$$

12. If possible, photocopy the following diagram for learners. It is a good summary of a frequency polygon. Alternately, learners should copy it in their exercise books.



13. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

14. Give learners an exercise to complete on their own.

15. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=cOgLCMQNVNU

https://www.youtube.com/watch?v=bEakGC6Ft0M

TERM 4, TOPIC 1, LESSON 3

CUMULATIVE FREQUENCY CURVES (OGIVES)

Suggested lesson duration: 2,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners should be able to: • populate or complete a frequency table • draw a cumulative frequency curve • read information from a cumulative frequency curve.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw the table from point 1 on the board (and ensure there is space to add two columns to it).
5. The table below provides references to this topic in Grade 11 textbooks. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		VIA AFRIKA		CLEVER		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
2	316	1 (1.4-1.6)	299	3	309	13.4	453	11.3	454

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. As with all lessons, prepare thoroughly. Ensure that all learners understand each concept. According to the diagnostic reports, learners find these concepts a challenge.
2. Even though each of the textbooks has an exercise on this, it is advisable to source another exercise or questions from a past test for learners to do as many questions as possible in the time available.

DIRECT INSTRUCTION

1. Ask learners to consider the following frequency table:

Mark	Frequency
11-20	5
21-30	9
31-40	14
41-50	16
51-60	12
61-70	9
71-80	5

2. Say: *This table represents the number of learners and their results on a test. Five learners got up to 20% but it is impossible to tell their exact results.*
3. Ask: *What is the modal class?*
   (41-50)
   Ask: *How many learners are represented?*
   (5 + 9 + 14 + 16 + 12 + 9 + 5 = 70)
   Ask: *What was the highest mark achieved in this test?*
   (80)
4. Tell learners to write the table in their books. They need to allow space to extend the table by two column as you look at different aspects of information. Learners should add to their table as you complete it on the board.
5. Ask: *How many learners got UP TO 20%?*
   (5). Fill this in on the first row of the cumulative frequency column.

Ask: *What does cumulative mean?*
(Accumulated/ gathered together).
Say: *In the cumulative frequency column, we are going to accumulate the totals.*
Listen carefully to the key words: UP TO.
Ask: *How many learners got UP TO 30%*
(14)

> If any learners said '9', explain why it is 14 – remind them you asked how many learners got UP TO 30% and the 5 learners who got up to 20% also belong in the category 'up to 30%'.

Fill in each accumulated frequency by stopping and asking learners for each one.
Show learners how they should look at the cumulative frequency in the row above and add the new amount in the next row to get their new total.

Mark	Frequency	Cumulative frequency
11-20	5	5
21-30	9	14
31-40	14	28
41-50	16	44
51-60	12	56
61-70	9	65
71-80	5	70

6. Say: *Notice that the last number filled in is the same as you gave me when I asked you how many learners' results were represented here.*
7. Ask: *How many learners achieved up to 50%?* (44)
   How many learners achieved up to 70%? (65)
   How many learners achieved up to 40%? (28)
8. Ask: *How many learners achieved MORE THAN 50%?* (26)
   Stop to ask what needed to be done now that we needed to focus on what comes after the accumulated total – we needed to subtract from the final accumulated total.
9. Ask: *How many learners achieved MORE THAN 20%?* (65)
   How many learners achieved MORE THAN 70%? (5)

> Continue to ask more of this type of questions if many learners are still finding it a challenge.

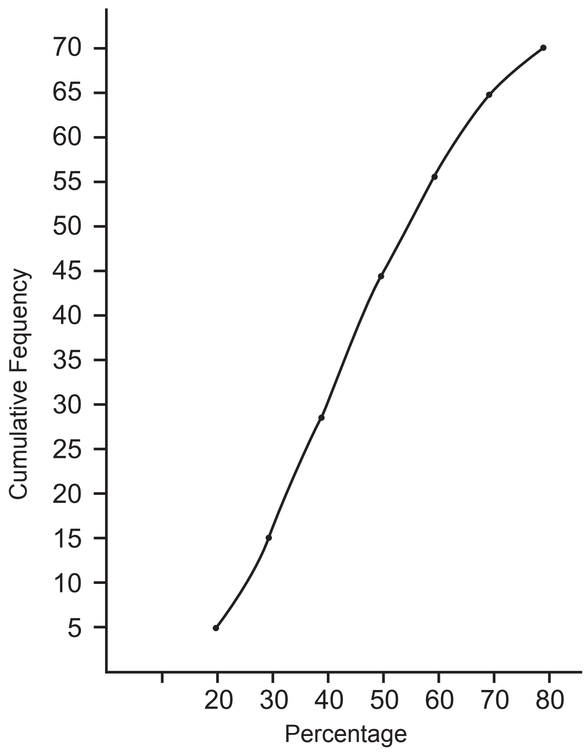
10. Say: *Before we draw a graph of this information, consider the following:*
    The accumulated amounts never went down. Even if one of the rows had represented zero learners, the total would have stayed the same.
    When we draw the graph, the total can never go down. This means that the graph should never go down either. Let's have a look at the graph together.

11. Firstly, we need to be sure what the co-ordinates are.
    Say: Remember, how I kept stressing 'UP TO'? This is a key part to remembering which numbers are important in the co-ordinates. The main co-ordinates are made up of the 'back' number in the interval and the accumulated frequency – both representing 'up to'.

12. Go back to the table on the board and add a further column. Highlight the upper boundary values in the intervals as well as the accumulated frequencies.

Mark	Frequency	Cumulative frequency	Co-ordinates
11-20	5	5	(20;5)
21-30	9	14	(30;14)
31-40	14	28	(40;28)
41-50	16	44	(50;44)
51-60	12	56	(60;56)
61-70	9	65	(70;65)
71-80	5	70	(80;70)

13. Before looking at the final co-ordinate, which needs an explanation of its own, draw the cumulative frequency curve with learners. Tell learners that a cumulative frequency is also called an ogive.
    Point out the following as you are drawing:
    - The horizontal axis will represent the data – in this case percentages.
    - Only the upper boundary numbers will be represented – show them all. These are the x-co-ordinates of the points found.
    - The vertical axis will always represent the cumulative frequency – no matter what situation is represented.
    - To choose a reasonable scale, take the highest number and divide by 10 – this gives an idea what multiples to use. In this case 70 ÷ 10 = 7. Rather use 5 or 10. We will use 5.
    - Plot the points and join them freehand and as smoothly as possible – remember it is called a cumulative frequency curve.

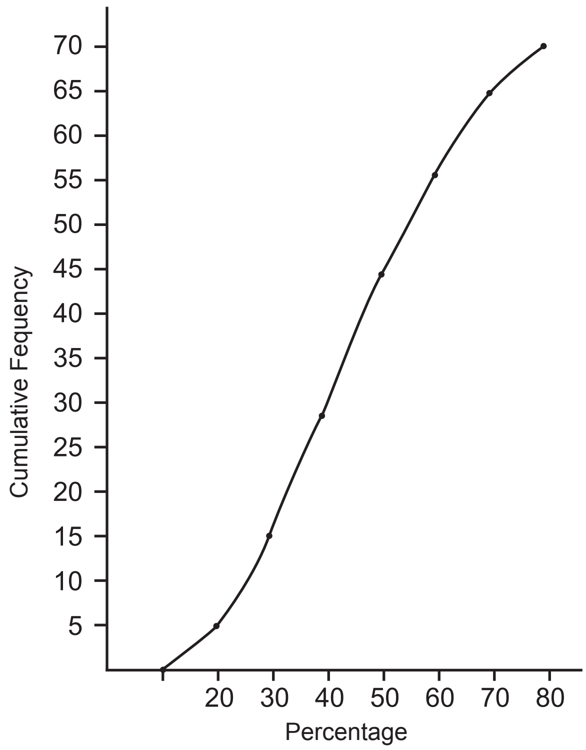


14. Once the main co-ordinates have been plotted and the ogive has been drawn, ask learners for their attention.

15. Learners should note that the first point is 'in the middle of nowhere'. The ogive needs to be grounded. The ogive is grounded to indicate that there are no values in the data set that are lower than the lower boundary of the first class interval.

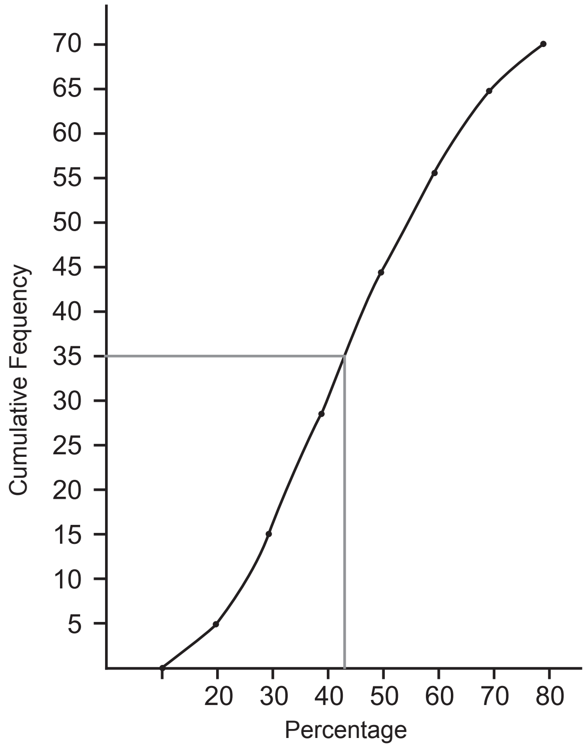
16. The co-ordinate for the grounding of the ogive is always the lower boundary of the first class interval (x-co-ordinate) and zero (y-co-ordinate). In this case, (11 ; 0).

17. Learners should plot the point and ground the ogive.



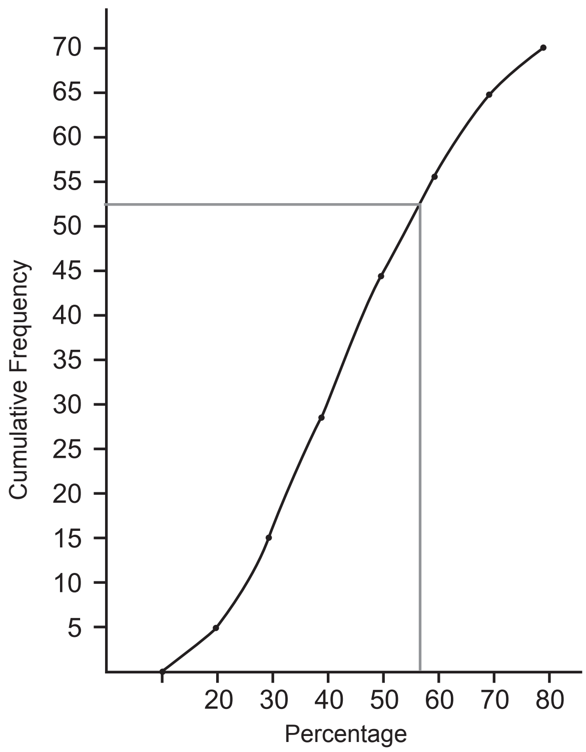
18. Learners should note the S-shape. An S-shape is common for an ogive.

19. Discuss what could be read from this visual representation.
    Show each point made on the ogive:
    - We could find an estimate of the median result. To do this, we need to know how many learners' results were in the data set. This will always be found on the vertical axis as it represents the cumulative frequency and therefore also shows the total.
    - As the total is 70, the median would be the 35^{th} learner. Mark 35 on the vertical axis and draw a horizontal line until it touches the ogive. Drop a vertical line from there to the horizontal axis and read off the percentage.



The median result is approximately 43% or 44%

- An estimate of the upper quartile could also be found. $\frac{3}{4}$ or 75% of 70 is 52,5. Mark this on the vertical axis and repeat the process described above.



The upper quartile is approximately 57%.

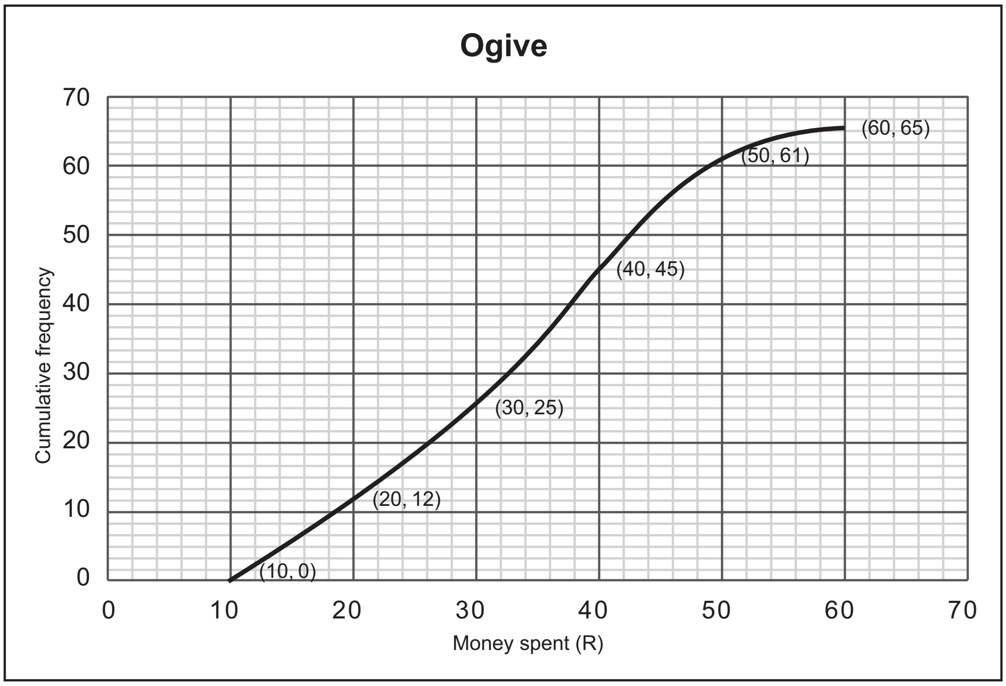
- Discuss the meaning of these statistics:
  - If the median is 43%, we can say that half of the learners scored below 43% and half scored above 43%.
  - If the upper quartile is 57% this means that three quarters of the learners scored below 57% and one quarter of learners scored above 57%.

> You may want to do a few more examples with learners.
> For example, find the the lower quartile and the 90th percentile.
> The estimated answers are: LQ – 30% and 90th percentile – 68%.

20. Once you feel learners are ready, do the following fully worked examples from past examinations with them.

Example 1

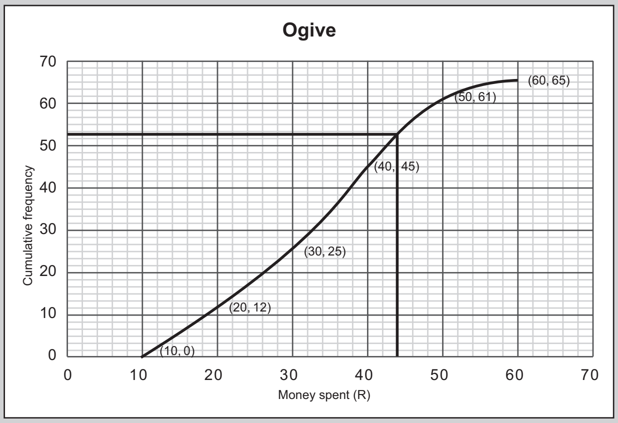
The amount of money, in rands, that learners spent while visiting a tuck shop at school on a specific day was recorded. The data is represented in the ogive below.



An incomplete frequency table is also given for the data:

Amount of money (in R)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency	a	13	20	b	4

a) How many learners visited the tuck shop on that day?
b) Write down the modal class of this data.
c) Determine the values of a and b in the frequency table.
d) Use the ogive to estimate the number of learners that spent at least R45 on the day the data was recorded at the tuck shop.

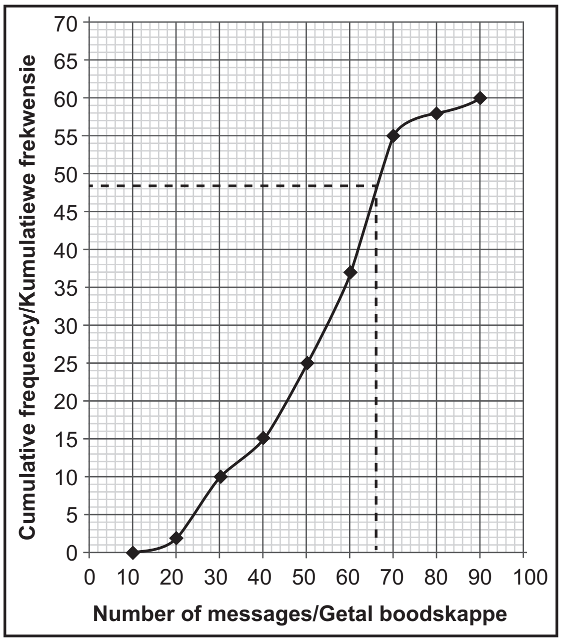
Solutions:	Teaching notes
a) 65 learners	This is the total represented in the last co-ordinate.
b) $30 \leq x < 40$	Tell learners that this can usually be seen by the part of the curve (from one co-ordinate to the next) that increases the most quickly. It is safer however, to look at each of the y-co-ordinates and calculate which interval has the most data.
c) $a = 12$ $b = 61-45$ $=16$	The value of a can be easily read from the co-ordinate (20;12) as no values have been accumulated yet. The value of b requires a subtraction calculation: the accumulated amount at the end of that interval subtract the accumulated amount at the end of the previous interval.
d) 11 or 12	Note the reading is at approximately 53 or 54. There are 65 learners in total, therefore 65 – 53(54) = 12(11) 

Example 2

A company recorded the number of messages sent by e-mail over a period of 60 working days. The data is shown in the table below:

NUMBER OF MESSAGES	NUMBER OF DAYS
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

a) Estimate the mean number of messages sent per day, rounded to two decimal places.
b) Draw a cumulative frequency graph (ogive) of the data on the grid.
c) Hence, estimate the number of days on which 65 or more messages were sent.

Solutions:	Teaching notes
a) $\frac{3\,080}{60} = 51{,}33$	Ask: *How do we find an estimated mean?* (Find the midpoint of the interval, multiply it by the frequency, total the frequencies and divide by 60).
b) 	Remind learners: ● To first make a cumulative frequency column ● 'Ground' the ogive. The first co-ordinate is always the lower part of the lowest boundary and 0. In this case (10;0) ● All the other co-ordinates are made up of the upper part of each boundary with the corresponding cumulative frequency. ● Join the points freehand – it should resemble a curve.

c) 60 – 48 = 12 days	Find 65 on the horizontal axis representing the number of messages. Read off the corresponding number on the vertical axis (cumulative frequency). Subtract this reading from the total as it said, ‘or more’.

21. Ask directed questions so that you can ascertain learners’ level of understanding. Ask learners if they have any questions.
22. Give learners an exercise to complete on their own.
23. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=sBK_oE8KDx8
(Drawing an ogive)

TERM 3, TOPIC 1, LESSON 4

VARIANCE AND STANDARD DEVIATION

Suggested lesson duration: 2,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners should be able to: ● explain what standard deviation means ● find the standard deviation from a set of data (using a calculator) ● comment on and interpret the standard deviation of a set of data.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw a simplified diagram of the five dogs.
5. The table below provides references to this topic in Grade 11 textbooks. Work through the lesson plan and decide where you will get learners to do the exercises.
   Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**VIA AFRIKA**		**CLEVER**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3	322	2	305	4 5&6 7	311 313 314	13.5	466	11.4	460

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Standard deviation is not an easy concept. Use the masses of the five dogs example. Work through the explanation methodically to ensure that no learner is left behind.
2. Avoid having learners do the calculations without understanding what they are finding. Encourage discussion and questions whenever possible.
3. Note that when calculator work is discussed, the Casio (80+ range) has been used. The diagnostic reports recommend using one brand on a regular basis to get used to the operation procedures. If more learners in your class have a brand other than the one being used, you must ensure they are confident using their own calculator.

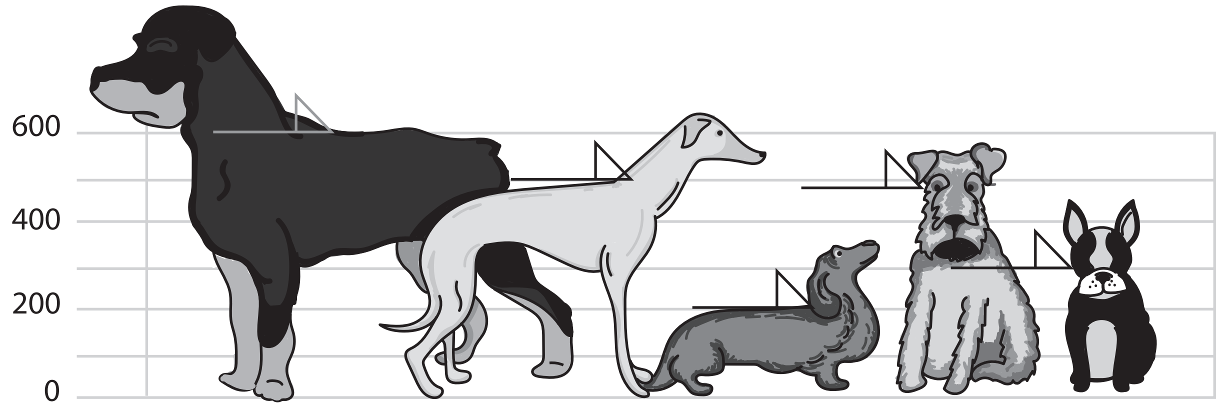
DIRECT INSTRUCTION

1. Start the lesson by saying: *We are going to look at a new concept: standard deviation. We will look at what standard deviation means, why we use it and how we find it.*
2. Tell learners that deviation means *'how far from the normal'*. The standard deviation is a measure of the spread of data. The symbol for standard deviation is σ, which is the lowercase form of the Greek letter, sigma (write the symbol on the board).
3. Variance is required to find standard deviation – so what does that mean? It is the average of the squared differences from the mean.

> Learners should write this definition down even though it may mean little at this point. You will refer to the definition again later.

4. The following example, using the heights of five dogs, will be used to further explain variance and standard deviation further. Tell learners what it is we are finding out about the data provided when we find standard deviation: we are finding what the norm is and which data lies within the norm and which data lies outside the norm.
   The heights of five dogs are found and recorded:

> This diagram is available in the Resource Pack. There is no need to draw it in detail on the board. Draw certain lines in as you go along with the example and explanation. If the diagram can't be copied, draw a basic representation. In addition, write the five heights clearly on the board.



Steps to follow:

- Find the mean of the heights.
- Find the difference between each dog's height and the mean (some answers will be negative).
- Square the differences.
- Find the average of the squared differences.
- Square root the answer.

Write these steps on the board but tell learners not to write them down yet as some of them need more explanation.
Learners should write the steps and make their own notes as you complete the example.

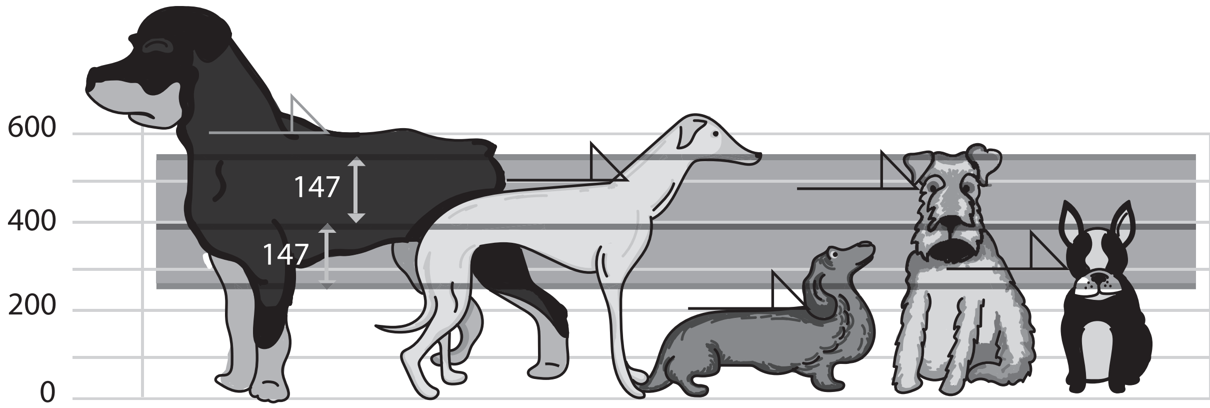
Height	Mean	Difference	Diff squared	Mean of squares
600mm	$\frac{1970}{5}$ = 394mm	206	42 436	$\frac{108\,520}{5}$ = 21 704
470mm		76	5 776	
170mm		-224	50 176	
430mm		36	1 296	
300mm		-94	8 836	
On the diagram: draw a horizontal line to represent the mean measurement.	Once the differences have been found: Ask: *Why do we need to square these numbers before we can find the mean of them?* (If we found the mean of a set of positive and negative integers it would not represent the data as the answer could even be quite close to zero).			

5. Remind learners that what we have found that 21 704 is the variance. Refer learners to the definition they wrote down earlier – variance is the mean of the squared differences. Point out that this very large number as it stands could not possibly tell us anything about how far each dog's height might be from the mean.

6. *Ask: Why do we square root this number to find standard deviation?*
   (We squared the differences to alleviate the problem of the negative integers. Finding the square root is the inverse operation of squaring a number. So, by finding the square root, we are reversing, or undoing, the squaring).

$$\sqrt{21704} = 147,322...$$

7. - Add 147mm to the mean (394 + 147 = 541)
   - Subtract 147mm from the mean (394 -147 = 247)
   - Draw a horizontal line at these two measurements. Shade the 'bar' created.



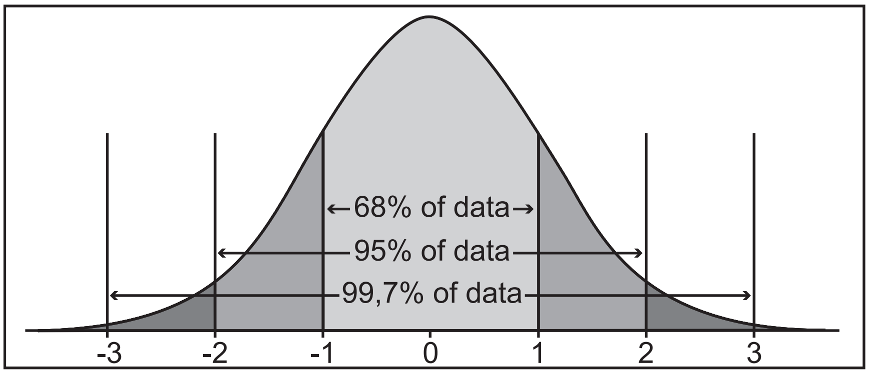
8. Say: *The shaded bar represents the heights within one standard deviation from the mean.*
   Repeat the statement and ensure that learners write it down.
   This tells us that, after taking all the data into account, we can see which dogs fall within one standard deviation of the mean and which dogs are considered 'outside the norm' and are either very tall or very short.

9. Show that we could make a wider bar if:
   - we added the standard deviation again to the top of the bar (541) to get 688
   - we subtracted the standard deviation again from the bottom of the bar (247) to get 100.

   If we drew in the horizontal lines representing the 688 and 100, we would now be seeing which dogs lay within TWO standard deviations from the mean.

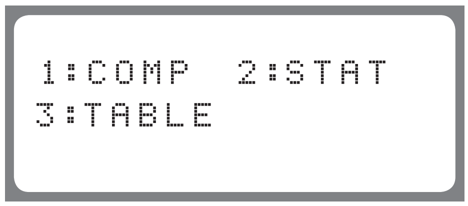
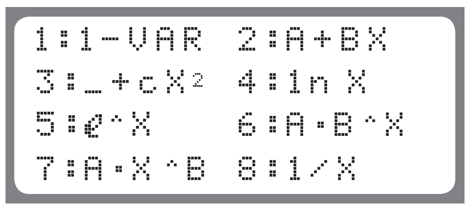
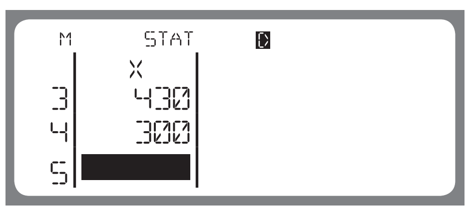
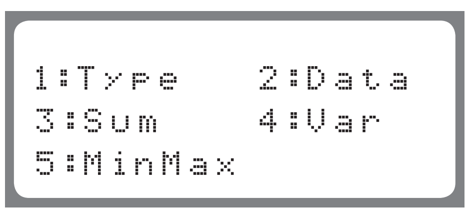
10. Explain the concept of standard deviation further by discussing what is considered to be the norm in a set of data:
    - 66% should lie within one standard deviation from the mean
    - 95% should lie within two standard deviations from the mean
    - 99,7% should lie within three standard deviations from the mean.

    Point out that this is only likely to be true if the set of data is large enough.
    The example we used above would not be considered sufficient data to draw any realistic conclusions.
    Learners may benefit from seeing this information visually and writing it in their books.



11. Show learners how to find standard deviation on the calculator.

'What you should see' is primarily for your benefit. If possible share it with learners. Alternately, tell them what they expect to see at each step.

Steps to follow	What you should see (dog heights used)
MODE, choose STAT (2)	
Choose 1-VAR (1)	
Enter data, pressing = after each number	
Press: AC; SHIFT, STAT (1)	
Choose Var (4)	

<table>
<tr><td>Choose σx (3)</td><td>1:n 2:$\overline{x}$
3:σx 4:sx</td></tr>
<tr><td colspan="2">Note: If there are questions relating to the data and what is (or isn't) within one standard deviation of the mean, then it is essential to know the mean.
After choosing VAR(4) or σx (3), press AC, shift, STAT again then choose $\overline{x}$ (2) to find the mean.</td></tr>
<tr><td>Press equal</td><td>σX
0

σX
147.3227749</td></tr>
</table>

12. Write the steps for learners. Ask learners to confirm their understanding by entering the five dog heights and confirming they get 147,322…

13. Learners should only use the long method if specifically asked. If this does occur, which is very rare, there is usually a table to complete (similar to the one completed in point 5).

    If variance is required in a question, learners will find the standard deviation then square it.

14. Give learners the opportunity to practice the process on their calculators. Give learners the following three sets of data. Ask learners to use the sets of data to find the standard deviation only. Go through all three sets of data with learners to ensure that they are comfortable with the process of using their calculator to find standard deviation. For each set of data, ask learners to list the data within one standard deviation from the mean for each set.

> Ideally, learners should work with data that represents a realistic situation that can be analysed further and lead to an understanding of the situation involved. However, this exercise is mainly to practice calculator work.

A	B	C
58	105	5,23
61	142	4,19
48	151	4,06
51	146	6,32
64	158	4,89
72	164	5,65
67	168	6,04
45	125	7,31
71	196	4,5
85	142	5,12
35	161	6,04
73	155	3,89

Solutions:

	Mean	Std Dev	Boundaries for one standard deviation	Data within one standard deviation from the mean
A	60,83	13,527	47,303 to 74,357	58; 61; 48; 51; 64; 72; 67; 71; 73
B	151,08	21,654	129,426 to 172,734	142; 151; 146; 158; 164; 168; 142; 161; 155
C	5,27	0,995	4,275 to 6,265	4,89; 5,65; 6,04; 4,5; 5,12; 6,04

15. Once learners have had the opportunity to become comfortable with the process on their own calculators, do the following fully worked example from a past examination.

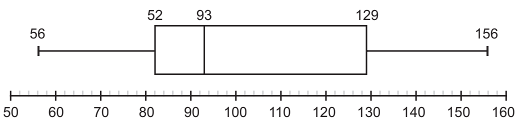
16. Say: *When we do this question, you will see that one question rarely just asks about standard deviation. Many other questions that relate to the spread of the data of which standard deviation is only one part may be asked.*

The table below shows the mass (to the nearest kilogram) of each of the 27 participants in a weight loss programme.

56	68	69	71	71	72	82	84	85
88	89	90	92	93	94	96	97	99
102	103	127	128	134	135	137	144	156

a) Calculate the range of the data.
b) Write down the mode of the data.
c) Determine the median of the data.
d) Determine the interquartile range of the data.
e) Draw a box-and-whisker diagram for the data.
f) Determine the standard deviation of the data.
g) The person weighing 127kg states that she has a mass of more than one standard deviation from the mean. Do you agree with this person? Motivate your answer with calculations.

NOV 2015

Solutions	Teaching notes
a) 156 - 56 = 100 b) 71 c) 93 d) $Q_3 - Q_1$ = 127 – 82 = 45	Questions a) – d) should be quite easy for learners at this stage.
e) 	Ask: *What statistics do we need to draw a box-and-whisker plot?* (The 5-number summary: lowest and highest values, lower and upper quartiles and median) Remind learners of the importance of drawing it to scale.
f) σ = 25,84	Calculator work.

g) $\bar{x}$ =98,59 Boundaries: 98,59 – 25,84 = 72,75 98,59 + 25,84 = 124,43 127 > 124,43 ∴ she is correct.	Ask: *What statistic do we need to answer this question?* (The mean) *What will we do once we have the mean and standard deviation?* (Find the lower an upper boundaries of one standard deviation by adding and subtracting the standard deviation from the mean).

17. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.
18. Give learners an exercise to complete on their own.
19. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=WVx3MYd-Q9w
(Calculating standard deviation)

https://www.youtube.com/watch?v=qqOyy_NjflU
(How to calculate variance and standard deviation)

TERM 4, TOPIC 1, LESSON 5

SYMMETRIC AND SKEWED DATA AND IDENTIFICATION OF OUTLIERS

Suggested lesson duration: 2,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	39

Lesson Objectives

By the end of the lesson, learners should be able to:

- interpret a set of data or its visual representation to comment on whether the data is skewed or not
- identify outliers in a set of data.

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson draw the histogram (point 3).
5. The table below provides references to this topic in Grade 11 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		VIA AFRIKA		CLEVER		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
4 5 6	324 329 333	3	310	8 9	317 319	13.2 13.3	442 447	11.5 11.6	463 466

CONCEPTUAL DEVELOPMENT

C

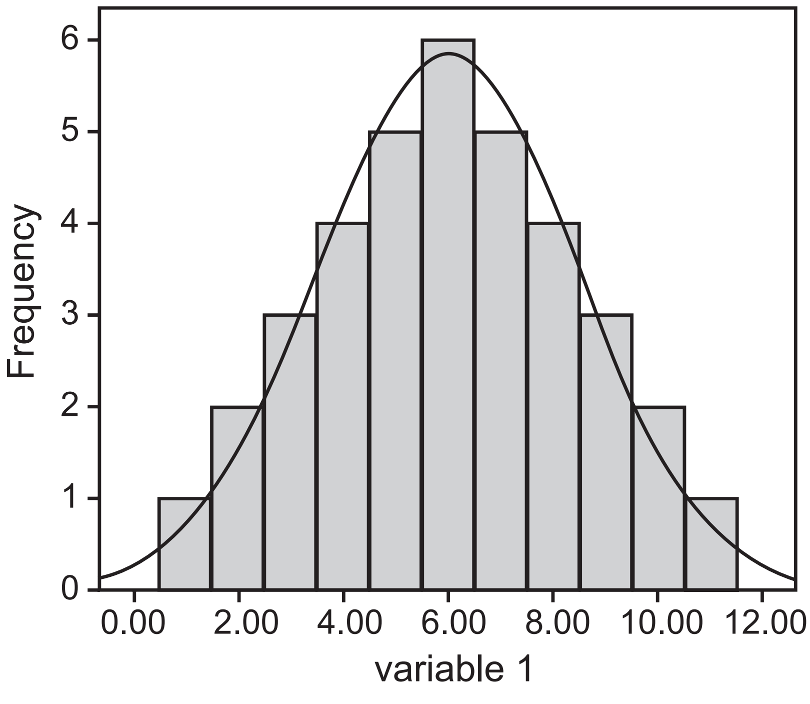
INTRODUCTION

1. Learners need to learn about skewness in a variety of contexts, not just in a box-and-whisker diagram.
2. In this lesson, we consider several ways of learning about skewness.

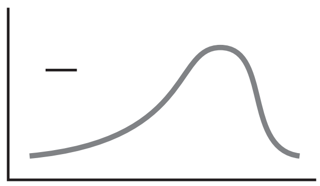
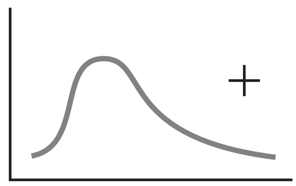
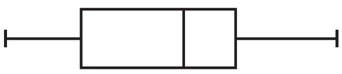
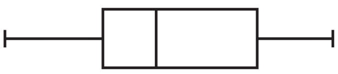
DIRECT INSTRUCTION

1. Tell learners they are going to learn about skewed data and that they should write the heading in their books. Remind learners to take notes as you explain, define or draw diagrams.
2. In general, data is skewed if there are outliers – data that are not part of the norm according to the rest of the data. Outliers are values that are significantly higher or lower than the rest of the data.

   If a histogram of a set of data looks as follows, then the data represented is said to be normally distributed. The mean and median will be equal (if the data is perfectly distributed) or very close to each other.



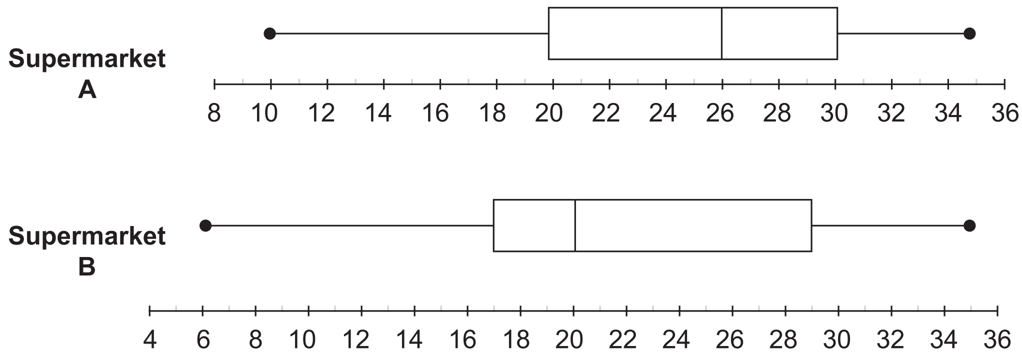
4. Write the following summary on the board for learners to write in their books. Discuss each aspect as you go along.

Skewed data		Teaching notes
Negatively skewed (mean subtract median is negative)	Positively skewed (mean subtract median is positive)	If the mean and median are known and there is no visual representation of the data, this method can be used to find in which direction the data is skewed
mean < median < mode Mean will be to the left of the median	mode < median < mean Mean will be to the right of the median	
		If a histogram or distribution curve is given (remind learners that the curve is a representation of the histogram), the 'tail' will show in which direction the data is skewed.
(Longer tail on left = skewed to left)	(Longer tail on right = skewed to the right)	
		If a box-and-whisker plot is available, the longer box will show in which direction the data is skewed. Tell learners to shade the longer part of the box and write 'skewed left' and 'skewed right' in the appropriate diagram.
Skewed to the <u>left</u> – the data is more spread out on the left	Skewed to the <u>right</u> – the data is more spread out on the right.	

6. Finish the discussion on skewed data by pointing out that:
   - the mean is susceptible to the influence of outliers and is not always a good representation of the data
   - both the mean and median are good representations of the data if the sample is normally distributed
   - if the data is skewed, the mean tends to be 'dragged' in the direction of the skewness – in this case the median would be a better measure of central tendency
   - the more skewed the data, the greater the difference between the mean and the median.
7. Ask: *Does anyone have any questions before we do some fully worked examples together?*

8. Once any questions have been answered, do the following examples from past examinations in full on the board. Learners should take them down in their books.

The number of delivery trucks making daily deliveries to neighbouring supermarkets, Supermarket A and Supermarket B, in a two-week period are represented in the box-and-whisker diagrams below.

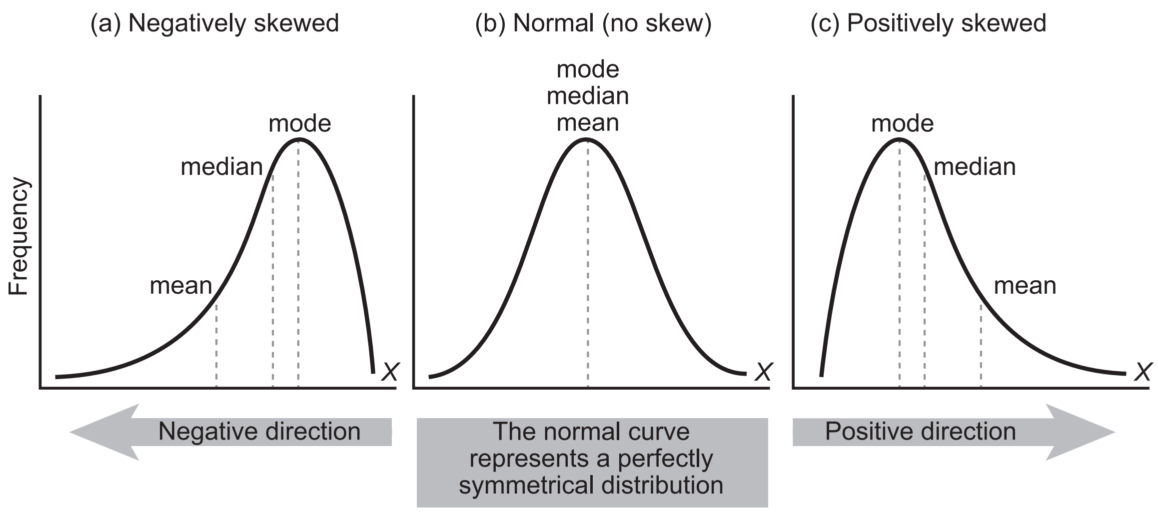


a) Calculate the interquartile range of the data for Supermarket A.
b) Describe the skewness in the data of Supermarket A.
c) Calculate the range of the data for Supermarket B.
d) During the two-week period, which supermarket receives 25 or more deliveries per day on more days? Explain your answer.

NOV 2014

Solutions	Teaching notes
a) $IQR = Q_3 - Q_1$ $= 30 - 20 = 10$ b) Skewed to the left (or negatively skewed). c) Range: 35 – 6 = 29	Questions a) – c) should be easy for learners to answer.
d) Supermarket A. For Supermarket B, the median is 20 which means 25 is in the upper half. For Supermarket A, the median is 26 which means 25 is not in the upper half, therefore, on more than half of the days, there are 25 deliveries.	Discuss this question with learners as they may find the interpretation confusing. Points to discuss: The data is split into quarters and if 25 lies in the 2nd quarter then the entire 3rd and 4th quarter (as well as part of the 2nd quarter) must be greater than 25 If 25 lies in the 3rd quarter, then only the entire 4th quarter and only part of the third quarter must be greater than 25.

9. Use the following sketches to discuss skewness of data:



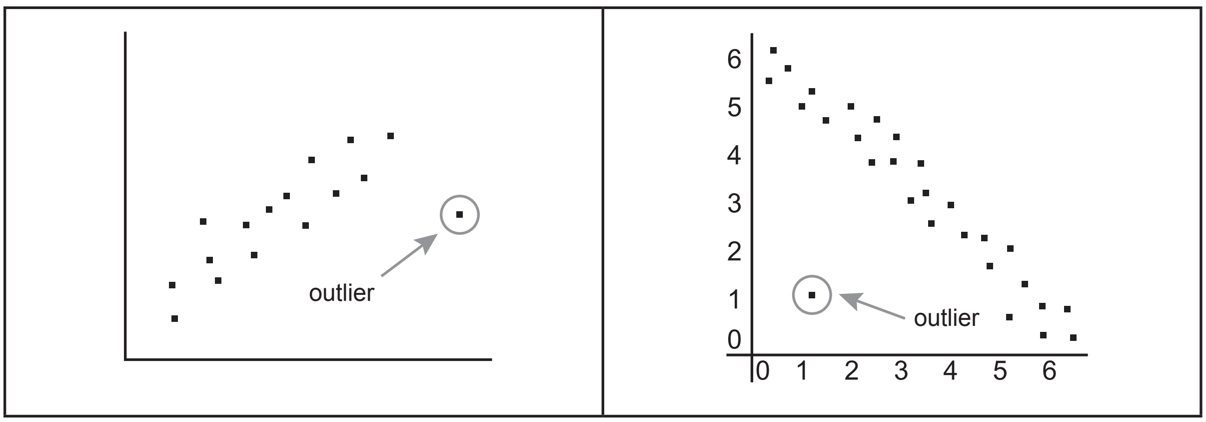
10. Say: *Now we need to look at the data that causes a set of data to be skewed.*
Ask: *What do we call the values that cause data to be skewed?*
(Outliers).

11. Tell learners to write the definition of outliers in their books:
Outliers are values that are significantly higher or lower than all the other values in the data set. Outliers are also called extremes.

12. In general, outliers can be found in two ways:
    - If the data is represented in a scatterplot, any outliers are usually quite clear.
    - If only the list of data is given, there is a calculation that can be done.

13. Say: *First we will look at some scatterplots to discuss outliers.*

14. Draw the following scatterplots on the chalkboard:



15. Point out the outliers in each of the diagrams.
    Say: *Note how obvious an outlier is when the data is represented in a scatterplot.*

16. Say: *The second way to find an outlier is by doing a calculation. The calculation requires finding the lower quartile, upper quartile and inter quartile range.*

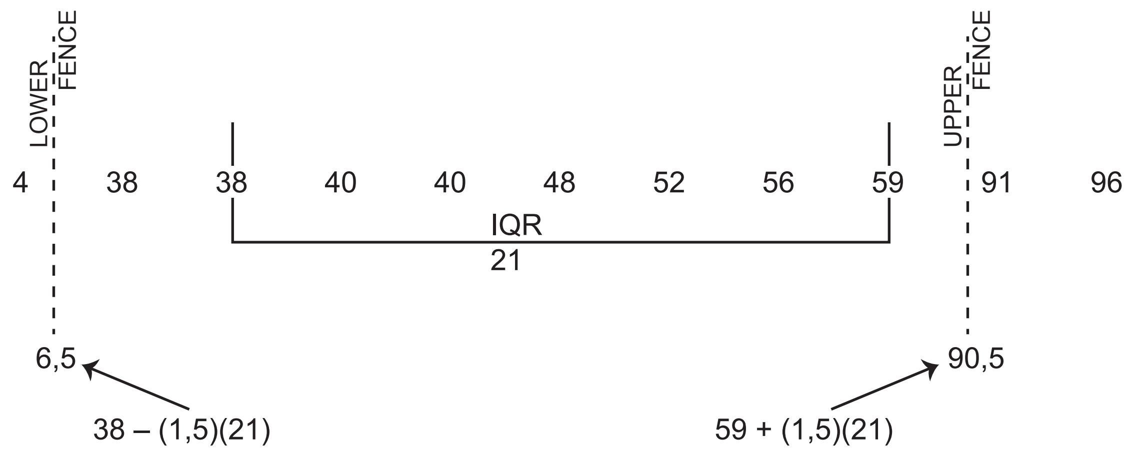
17. Write the following steps on the chalkboard:
    - Find the lower quartile and upper quartile
    - Find the inter quartile range
    - Multiply the interquartile range by 1,5
    - Subtract this number from the lower quartile and add it to the upper quartile
    - If any data lies outside these two numbers, they are outliers.

18. Once learners have written the steps in their books, do the following example with them.

Look at the following data set for outliers:

4   30   38   40   40   48   52   56   59   91   96

Lower quartile: 38
Upper quartile: 59
IQR: 59 – 38 = 21
21 × 1,5 = 31,5
38 – 31,5 = 6,5   59 + 31,5 = 90,5
There are 3 outliers in this set of data: 4, 91 and 96



19. Tell learners that the two numbers found can be called a lower fence and an upper fence – implying that only the data inside the fence can be relied on.

20. Write the following on the chalkboard to show learners how outliers can change the way a set of data looks:

Without Outlier	With Outlier
4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7	4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 300
Mean = 5.45	Mean = 30.00
Median = 5.00	Median = 5.50
Mode = 5.00	Mode = 5.00
Standard Deviation = 1.04	Standard Deviation = 85.03

Say: *Notice how the median is still much the same but that the mean and standard deviation are very different in the data sets with and without the outlier.*

21. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

22. Give learners an exercise to complete on their own.

23. Walk around the classroom as learners do the exercise. Support learners where necessary.

D ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=XSSRrVMOqIQ
(What is skewness)

https://www.youtube.com/watch?v=mwT3ykS8r08
(Skewed data and outliers)

https://www.youtube.com/watch?v=n5rhuZDbYCM
(Determining skewness in ogive curves)

https://www.youtube.com/watch?v=9aDHbRb4Bf8
(Identifying outliers)

TERM 3, TOPIC 1, LESSON 6

REVISION AND CONSOLIDATION

Suggested lesson duration: 1,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners will have revised: • all the concepts required in this topic.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. The table below provides references to this topic in Grade 12 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		**PLATINUM**		**VIA AFRIKA**		**CLEVER**		**EVERYTHING MATHS (SIYAVULA)**	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some Ch	334 336	Rev	313	Qu’s	320	Rev	472	11.7	468

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Ask learners to recap what they have learned in this section. Point out issues that you know are important as well as problems that you encountered from your own learners.

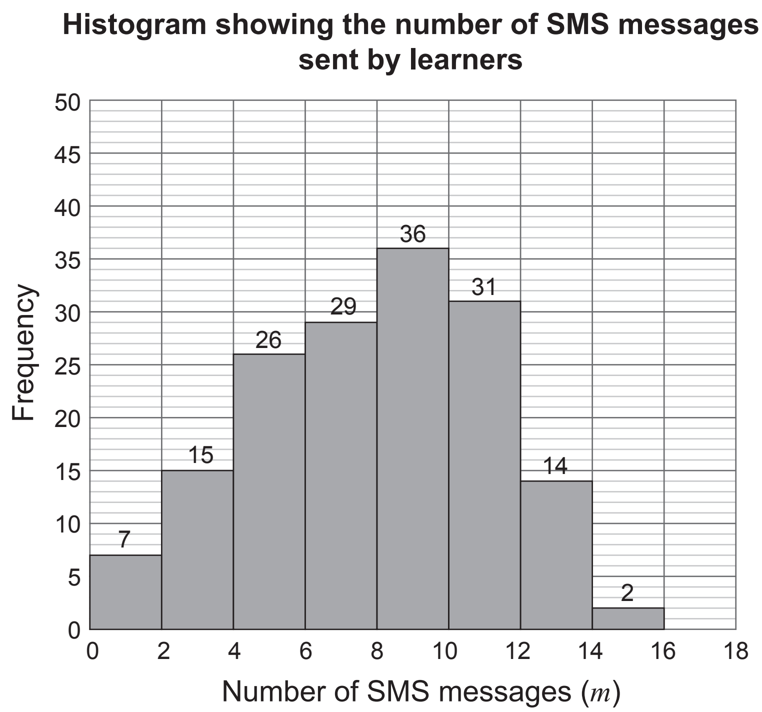
2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

1. Say: *Before you do the revision exercise on your own, we will do one final past examination question to cover as many aspects as possible from this section.*

2. Learners should work through this question with you. Encourage the learners to tell you what to do before doing it.

A group of Grade 11 learners were interviewed about using a certain application to send SMS messages. The number of SMS messages, m, sent by each learner was summarised in the histogram below.



a) Complete the cumulative frequency table.

CLASS	FREQUENCY	CUMULATIVE FREQUENCY
$0 \leq m < 2$		
$2 \leq m < 4$		
$4 \leq m < 6$		
$6 \leq m < 8$		
$8 \leq m < 10$		
$10 \leq m < 12$		
$12 \leq m < 14$		
$14 \leq m < 16$		

b) Use the table to draw a cumulative frequency curve (ogive).

c) Use the ogive to identify the median for the data.

d) Estimate the percentage of learners who sent more than 11 messages using this application.

e) In which direction is the data skewed?

Exemplar 2013

Solutions	Teaching notes

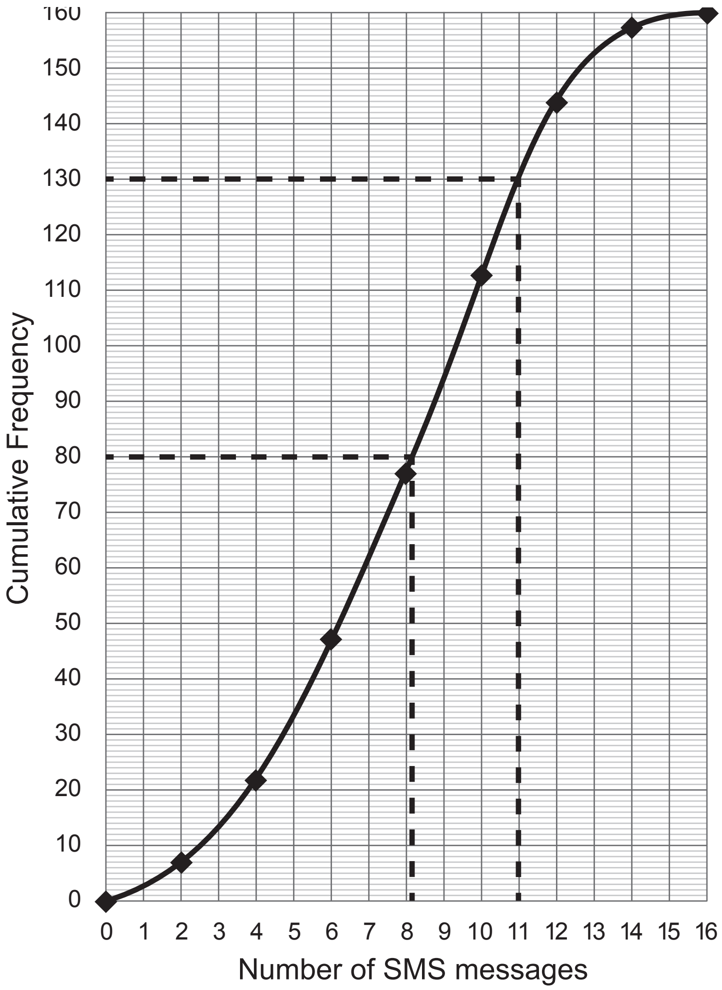
a) Learners should be able to do this without assistance. Remind them that they need to accumulate as they go.

CLASS	FREQUENCY	CUMULATIVE FREQUENCY
$0 \leq m < 2$	7	7
$2 \leq m < 4$	15	22
$4 \leq m < 6$	26	48
$6 \leq m < 8$	29	77
$8 \leq m < 10$	36	113
$10 \leq m < 12$	31	144
$12 \leq m < 14$	14	158
$14 \leq m < 16$	2	160

b) Ask:
What will the vertical axis represent?
(Cumulative frequency).
What is the total?
(160)
What scale could be used?
(Multiples of 10).
What will the horizontal axis represent?
(Number of SMS messages).
What is the highest value?
(16).
What scale could be used?
(Multiples of 1 or 2).
How do we find co-ordinates?
(Upper boundary with cumulative frequency).
What co-ordinate will ground the ogive?
(0;0)

c) Ask: *What is the total number of learners?*
(160).
NOTE: Learners often struggle with working out where (that is, which axis) they should read the measures of dispersion from. Instead of telling learners it is always the cumulative frequency, explain why.
Ask learners to think about this data being collected and to ask themselves where the data came from? In this case, from the learners – it would be their information 'lined up' to find the median, for example, so they therefore need to look at the axis representing all the learners.
Ask: *As the median is required, what is half of 160?*
(80).
Say: *Find that, draw a horizontal line to the ogive and a vertical line to the axis to read off the number of SMSs*.

d) Say: *The estimation of the percentage of learners who sent more than 11 message will be found in a similar manner to finding the number of SMSs sent, but in reverse. Mark off where 11 SMS's are represented; draw a vertical line to the ogive then a horizontal line to the axis to read off the number of learners.*
As we are dealing with 'more than' this needs to be subtracted from 160. Then, find the percentage.



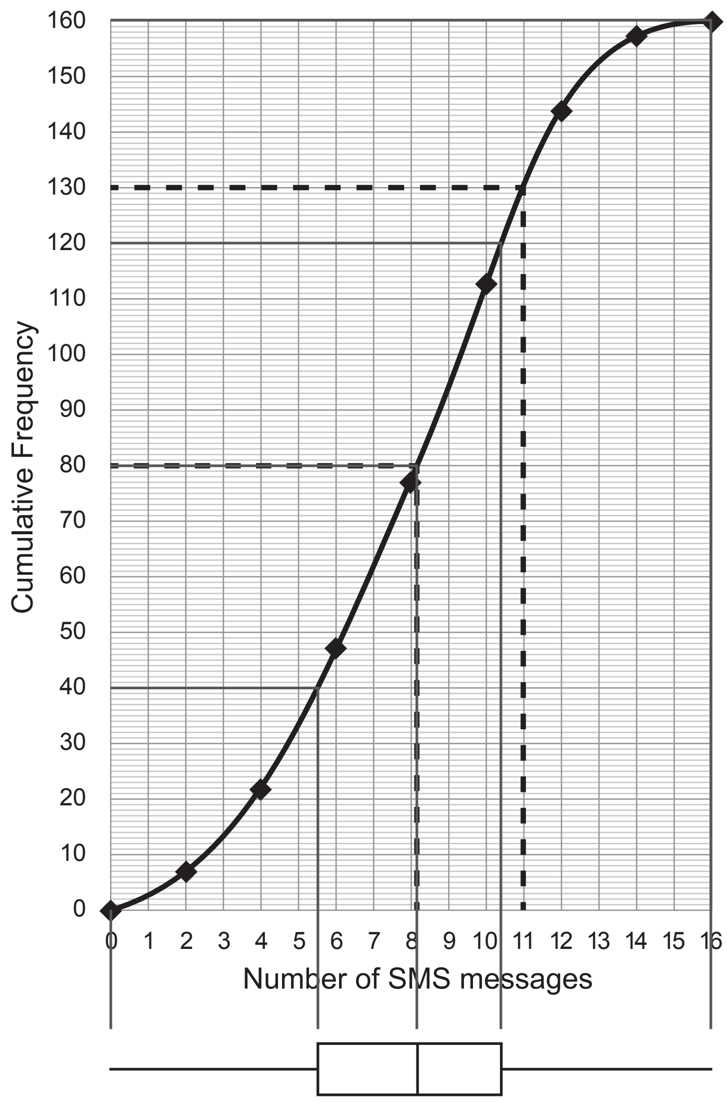
c) 8 messages

d) Reading: 130

∴ 160 - 130 = 30

30 learners sent more than 11 messages ∴ $\frac{30}{160} \times 100 = 18{,}75\%$

e) Say: *An excellent way to find skewness when given an ogive is to use the ogive to draw a box- and-whisker diagram. The median has already been marked off, so we need to find where the Lower Quartile and Upper Quartile are. Mark these the same way the median was found. Then mark the lowest and highest value and draw the box and whisker. The scale will be correct from the ogive.*



e) The data is slightly skewed to the left.

3. Ask learners to do the revision exercise from their textbook. If you have an extra worksheet or a past test paper, this would also be an excellent way for learners to consolidate what they have learned. It would also give them the opportunity of knowing what to expect when they must do an assessment.

4. Ask directed questions so that you can ascertain learners' level of understanding. Ask learners if they have any questions.

5. Walk around the classroom as learners do the exercise. Support learners where necessary. Stop learners at certain times to discuss or mark a question on the board. Use the revision time well.

Term 4

REVISION OVERVIEW

A. TOPIC OVERVIEW

A

- The revision plan runs for three weeks (13.5 hours).
- The revision plan is not presented over specific lessons. We provide guidance regarding what to complete in each week. Plan according to your own learners needs.
- Learners will write two examinations in November. Each examination is three hours and 150 marks. Encourage learners to be well prepared.
- It is essential that you use these revision weeks to the maximum benefit of the learners. Learners need to feel confident when writing their final examinations.
- The revision programme is made up of three parts:
    - Summary notes to share with learners
    - One full Paper 1 and Paper 2 (2017) to work through with learners in detail.
    - One Paper 1 and Paper 2 (exemplars) for learners to work on in class and at home and make 'cheat sheets' (their own summaries) at the same time.

Breakdown of revision programme:

Week 1	Paper 1 summary notes and past Paper 1
Week 2	Paper 2 summary notes and past Paper 2
Week 3	Paper 1 and Paper 2 + 'cheat sheets'.

As part of the revision programme, learners will work through past papers. This has been shown to be an excellent learner-centred approach to revision.

In addition to providing the past papers and memoranda in the Resource Pack, we provide the following links:

Links for past papers and memoranda:	
Paper 1 2017	http://www.edwardsmaths.com/wp-content/uploads/2018/01/NSC-GR11-MATHEMATICS-P1-NOV2017-FINALS-ENGLISH.pdf
Paper 1 2017 memo	http://www.edwardsmaths.com/wp-content/uploads/2018/01/Final-Marking-Guideline-Grade-11-Mathematics-P1-2017-Common-Examination.pdf
Paper 2 2017	http://www.edwardsmaths.com/wp-content/uploads/2018/01/NSC-GR11-MATHEMATICS-P2-NOV2017-FINALS-ENGLISH.pdf

Paper 2 2017 memo	http://www.edwardsmaths.com/wp-content/uploads/2018/01/NSC-GR-11-Maths-P2-Memo.pdf
Paper 2 2016	http://www.edwardsmaths.com/wp-content/uploads/2016/12/Mathematics-P2-Grade-11-Nov-2016-Eng.pdf
Paper 2 2016 memo	http://www.edwardsmaths.com/wp-content/uploads/2016/12/GRADE-11-MATHEMATICS-P2-memo.pdf
Paper 1 Exemplar 2013	https://www.ecexams.co.za/2013_Exemplars.htm (These four documents can be found in a folder on the site from this link)
Paper 1 Exemplar 2013 memo	
Paper 2 Exemplar 2013	
Paper 2 Exemplar 2013 memo	

B WHAT EXPERIENCE AND RESEARCH TELLS US ABOUT PREPARING FOR EXAMINATIONS

WHAT IS THE BEST WAY TO REVISE FOR A MATHS EXAM?

- Learn your theory.
- Do practice questions.
- Check your answers.
- Focus on what you can do, as well as what you can't do.
- Discuss questions and methods with fellow learners. Explain to each other – this is an excellent way to consolidate your own understanding.
- Make a revision plan and stick to it.

KEEP YOUR EYE ON THE PRIZE - SHARE THESE TIPS WITH YOUR LEARNERS

Revising for maths exams can be hard work – it can mean making sacrifices where you choose to prioritise revision over other things. Therefore, it is always important to keep your eye on the prize. Think about what your maths qualification will mean for your future life and career. Hopefully this will keep you motivated when times are tough during revision.

ASSESSMENT

C

- CAPS formal assessment requirements for Term 4:
  - Test (already completed)
  - Final examinations (Paper 1 and Paper 2)
- The examinations will be made up as follows:

Paper 1

	Mark allocation
Number patterns	(25±3)
Algebraic Expressions, Equations and Inequalities	(45±3)
Functions	(45±3)
Finance and Growth	(15±3)
Probability	(20±3)

Paper 2

	Mark allocation
Trigonometry	(60±3)
Analytical geometry	(30±3)
Euclidean Geometry and Measurement	(40±3)
Statistics	(20±3)

TERM 4

REVISION - WEEK 1

A POLICY AND OUTCOMES

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners will have: ● worked through summaries of all Paper 1 topics ● completed a full Paper 1 in class with their teacher.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the summaries of Paper 1
   - work through the examination and teaching notes.
3. The notes and examination are both available in the resource booklet for photocopying if possible.
4. Write work on the chalkboard before the learners arrive to ensure no time is wasted.

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. Support learners as they consolidate all that they have learned this year.
2. Give learners time to ask questions and become confident in their ability to write their final examination.

DIRECT INSTRUCTION

1. Start the lesson by handing out Resource 7 in the Resource Pack. This resource contains summary notes for the topics assessed in Paper 1.
2. Work through the notes with learners. This should take at least an hour.
3. As you go through each topic, ask questions to ascertain how much learners remember.
4. Encourage learners to add their own notes to the summary notes you have given them.
5. Once you have revised each section (for Paper 1), hand out the past examination paper (This is Resource 8 in the Resource Pack – Paper 1, 2017). Work though each question in detail. Some learners may be sufficiently confident to work on their own, while others may prefer to work with you.
6. As you go through each question, give learners the opportunity to contribute and to ask questions.
7. Encourage learners to use their summary notes – for answering questions and to add their own notes as they go along.

ALGEBRA, EXPONENTS, SURDS, EQUATIONS AND INEQUALITIES

a) Solve for x:
   (i) $(2x - 3)(x + 7) = 0$
   (ii) $7x^2 + 3x - 2 = 0$ (leave your answer correct to TWO decimal places)
   (iii) $\sqrt{x-1} + 3 = x$
   (iv) $x^2 > 3(x + 6)$

b) Solve for x and y simultaneously:
   $2y + x = 1$
   $x^2 + y^2 + 3xy + y = 0$

c) If $f(x) = 0$ has roots $x = \dfrac{-5 \pm \sqrt{3 - 12k^2}}{4}$, for which values of k will the roots be equal?

Teaching notes:

a)

(i) A common error in this type of question is for learners to multiply out and factorise. Point out that the equation is already in factorised form and the solution can immediately be found.

(ii) When the number of decimal places is mentioned, it is usually a clue that the quadratic formula will be used. Point this out to learners.

(iii) When solving equations with surds remind learners of the following steps and points to remember:

- Use inverse operations to get the term with the surd on its own
- Square both sides
- Solve as usual from this step

BUT – remember to ALWAYS check each solution to ensure that both solutions are possible.

(iv) When solving quadratic inequalities, remind learners of the following steps and points to remember:

Get all terms on one side and factorise (as with quadratic equations)
State the critical values
Draw a sketch to represent the quadratic function
Using the inequality, note the matching part of the function (positive or negative)
State the solution using the correct inequality.

b)

Remind learners to make one variable the subject of the formula in one of the equations then to use this information to substitute into the other equation. Solve for the unknown variable and substitute back into the other equation to solve for the second variable.

c)

Ask: *For roots to be equal, what does $b^2 - 4ac$ need to equal?* (zero)

Tell learners to make $3 - 12k^2 = 0$ and solve for k.

Solutions:

a)

(i) $(2x - 3)(x + 7) = 0$

$x = \frac{3}{2}$ or $x = -7$

(ii) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(7)(-2)}}{2(7)}$

$x = \frac{-3 \pm \sqrt{65}}{14}$

$x = -0{,}79$ or $x = 0{,}36$

(iii) $\sqrt{x - 1} + 3 = x$

$\sqrt{x - 1} = x - 3$

$(\sqrt{x - 1})^2 = (x - 3)^2$

$x - 1 = x^2 - 6x + 9$

$0 = x^2 - 7x + 10$

$0 = (x - 5)(x - 2)$

$x = 5$ or $x = 2$ (N/S)

$\therefore x = 5$

(iv) $x^2 > 3(x+6)$

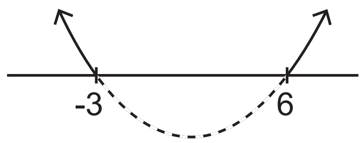
$x^2 - 3(x+6) > 0$

$x^2 - 3x - 18 > 0$

$(x-6)(x+3) > 0$

CV'S: 6 and - 3

$x < -3$ or $x > 6$



b) $2y + x = 1$ $\qquad x^2 + y^2 + 3xy + y = 0$

$x = 1 - 2y$

$(1-2y)^2 + y^2 + 3y(1-2y) + y = 0$

$1 - 4y + 4y^2 + y^2 + 3y - 6y^2 + y = 0$

$1 - y^2 = 0$

$(1-y)(1+y) = 0$

$y = 1$ or $y = -1$

$\therefore x = -1$ or $x = 3$

c) $3 - 12k^2 = 0$

$3(1 - 4k^2) = 0$

$1 - 4k^2 = 0$

$(1-2k)(1+2k) = 0$

$k = \frac{1}{2}$ or $k = -\frac{1}{2}$

a) Simplify fully without using a calculator:

$$\frac{3^{m+4} - 6.3^{m+1}}{7.3^{m+2}}$$

b) Solve for x without using a calculator:

(i) $x^{-\frac{3}{4}} = 8$

(ii) $4^x - 2^x = 2$

c) If $x = \frac{3-\sqrt{a}}{\sqrt{2}}$ and $y = \frac{4+\sqrt{a}}{\sqrt{2}}$, determine the value of $(x + y)^2$

d) Show, WITHOUT using a calculator that $\sqrt[12]{10} \times \sqrt[6]{640} \times \sqrt[4]{810} \times \sqrt{40} = 120$

Teaching notes:

a)

Remind learners: When the numerator and/or denominator have more than one term, you need to factorise. To find a common factor, use Law 1 in reverse ($3^{m+4} = 3^m . 3^4$). This makes it easier to find the HCF and to know what remains when it has been taken out.

b)

(i) Remind learners: If an equation is in the form: $x^{\frac{a}{b}} = y$

- there will be a positive and negative solution if a is even and b odd.
- there will be one solution if a is odd.

Therefore, we expect one solution in this case.

Ask: *What do we need to do to make x the subject of the formula?*

(Raise both sides to the inverse of the exponent).

(ii) Learners tend to find this type of questions a challenge. Remind learners to look out for three terms where one has an exponent that is two times (2 x) one of the other term's exponent. In this case, once the 4 has been written as a product of its prime factors it should be clear that one of the exponents is $2x$ and the other is x. This should lead to the use of the substitution method (k-method).

c)

Tell learners that this question may look complicated but essentially it is a multiplication of two fractions using the FOIL method (distributive law). The key is to know how to multiply with surds.

d)

The key to answering this question is to recognise the perfect squares that are factors of the numbers inside the root signs. Writing the numbers as a product of the perfect square and another factor then writing the perfect square as a product of its prime factors should lead to a straightforward simplification of the expression.

a) $\frac{3^{m+4} - 6.3^{m+1}}{7.3^{m+2}}$

$= \frac{3^m(3^4 - 6.3^1)}{7.3^m 3^2}$

$= \frac{(3^4 - 6.3^1)}{7.3^2}$

$= \frac{81 - 18}{63}$

$= \frac{63}{63} = 1$

b) (i) $x^{-\frac{3}{4}} = 8$

$x^{-\frac{3}{4}} = 2^3$

$x = (2^3)^{-\frac{4}{3}}$

$x = 2^{-4}$

$x = \frac{1}{16}$

(ii) $4^x - 2^x = 2$

$4^x - 2^x - 2 = 0$

$2^{2x} - 2^x - 2 = 0$

Let $2^x = k$

$\therefore k^2 - k - 2 = 0$

$(k-2)(k+1) = 0$

$k = 2$ or $k = -1$

$2^x = 2$ or $2^x = -1$

$\therefore x = 1$ $\therefore$ N/S

c) $(x+y)^2 = (\frac{3-\sqrt{a}}{\sqrt{2}} + \frac{4+\sqrt{a}}{\sqrt{2}})^2$

$= (\frac{3-\sqrt{a}}{\sqrt{2}} + \frac{4+\sqrt{a}}{\sqrt{2}})(\frac{3-\sqrt{a}}{\sqrt{2}} + \frac{4+\sqrt{a}}{\sqrt{2}})$

$= \frac{9-6\sqrt{a}+a}{2} + \frac{12-\sqrt{a}-a}{2} + \frac{12-\sqrt{a}-a}{2} + \frac{16+8\sqrt{a}+a}{2}$

$= \frac{49}{2}$

d) $\sqrt[12]{10} \times \sqrt[6]{640} \times \sqrt[4]{810} \times \sqrt{40}$

$= \sqrt[12]{10} \times \sqrt[6]{64.10} \times \sqrt[4]{81.10} \times \sqrt{4.10}$

$= \sqrt[12]{10} \times \sqrt[6]{2^6.10} \times \sqrt[4]{3^4.10} \times \sqrt{2^2.10}$

$= 10^{\frac{1}{12}}.2^1.10^{\frac{1}{6}}.3^1.10^{\frac{1}{4}}.2^1.10^{\frac{1}{2}}$

$= 10^1.2^2.3^1$

$= 120 = \text{RHS}$

NUMBER PATTERNS

a) Given the finite linear pattern: 12 ; 17 ; 22 ; …. ; 172

(i) Determine a formula for the nth term of the pattern.

(ii) Calculate the value of T_{12}

(iii) Determine the number of terms in the pattern.

b) Given the first four terms of a linear pattern: 3 ; x ; y ; 30

Calculate the values of x and y.

Teaching notes:

a)

Ask: *What is the key point of a linear pattern?*

(There is a common difference between the terms).

Ask: *What does 'finite' mean?*

(The pattern will come to an end. It does not go on indefinitely).

(i)

Say: *Find the common difference and use the formula to find the pattern.*

(ii)

Ask: *How do we find a certain term in a pattern?*

(Substitute the position given for 'n').

(iii)

Ask: *How do we find the position of a term?*

(Make the general term equal to the term itself and solve for 'n').

b)

Ask: *What is the key point of a linear pattern?*

(There is a common difference between the terms)

Say: *We need to use this knowledge to form equations in which we can solve for the unknown.*

a) (i) 12 17 22

5 5

$T_n = a + (n - 1)d$

$T_n = 12 + (n - 1)5$

$T_n = 5n + 7$

(ii) $T_{12} = 5(12) + 7 = 67$

(iii) $172 = 5n + 7$

$165 = 5n$

$n = 33$

b) 3 x y 30

$x - 3$ $y - x$ $30 - y$

$x - 3 = y - x$ and $30 - y = y - x$

$2x - 3 = y$

$\therefore 30 - (2x - 3) = 2x - 3 - x$

$30 - 2x + 3 = x - 3$

$-3x = -36$

$x = 12$

$\therefore y = 2(12) - 3 = 21$

Given the quadratic pattern: 244 ; 193 ; 148 ; 109 ; ….

a) Write down the next term of the pattern

b) Determine a formula for the nth term of the pattern.

c) Which term of the pattern will have a value of 508?

d) Between which two consecutive terms of the quadratic pattern will the first difference be 453?

e) Show that all the terms of the quadratic pattern are positive.

Teaching notes:

a)

Ask: *What can you tell me about a quadratic pattern?*

(There is a second common difference).

Say: *We will use this idea to work out the next term.*

Once the second common difference has been found, add 6 to final 1st difference found and add the answer to 109 to find the next term.

b)

Remind learners of the steps to find a quadratic pattern ($ax^2 + bx + c$):

To find a: make $2a$ equal to the second difference

To find b: make $3a + b$ equal to the first difference found (-51)

To find c: make $a + b + c$ equal to the first term.

c)

Ask: *How do we find the position of a term?*

(Make the general term equal to the term itself and solve for 'n').

Remind learners that this is a quadratic pattern and they will therefore find two solutions for n, and that they need to use the correct solution – the position of a term in a pattern must be a natural number.

d)

Say: *To do this, find the general term of the linear pattern found in the first line of differences then find the position of 453 in that pattern. The solution will be between that term and the following term.*

e)

Discuss what makes a quadratic positive. Remind learners of the relationship between a quadratic pattern and a quadratic function.

Ask: *If a quadratic function is positive, where would it be in the Cartesian plane?*

(Above the x-axis).

Remind learners that if the parabola is positive and has a turning point in quadrant 1 or 2 then this will make it always positive.

Say: *We will 'complete the square' on the quadratic function to verify that the turning point is in quadrant 1 or 2 which will show that this pattern is always positive.*

a) 244   193   148   109

–51   –45   –39

6   6

$T_5 = 76$

b) $2a = 6$

$a = 3$

$3a + b = -51$

$3(3) + b = -51$

$b = -60$

$a + b + c = 244$

$3 - 60 + c = 244$

$c = 301$

$T_n = 3n^2 - 60n + 301$

c) $508 = 3n^2 - 60n + 301$

$0 = 3n^2 - 60n - 207$

$0 = 3(n^2 - 20n - 69)$

$0 = (n + 3)(n - 23)$

$\therefore n = -3$ or $n = 23$

$n \neq -3 \therefore n = 23$

d) $T_n = a + (n - 1)d$ (1st difference)

$T_n = -51 + (n - 1)6$

$T_n = 6n - 57$

$453 = 6n - 57$

$510 = 6n$

$n = 85$

$\therefore$ between T_{85} and T_{86}

e) $T_n = 3n^2 - 60n + 301$

$T_n = 3(n^2 - 20n + \frac{301}{3})$

$T_n = 3(n^2 - 20n + 100 + \frac{301}{3} - 100)$

$T_n = 3[(n - 10)^2 + \frac{1}{3}]$

$T_n = 3(n - 10)^2 + 1$

$3(n - 10)^2 + 1 \geq 0$ for all values of n

$\therefore$ all terms in the pattern are positive

FUNCTIONS

Given: $f(x) = \frac{-3}{x+2} + 1$ and $g(x) = 2^{-x} - 4$

a) Determine $f(-3)$

b) Determine x if $g(x) = 4$

c) Write down the asymptotes of f

d) Write down the range of g

e) Determine the coordinates of the x and y - intercepts of f

f) Determine the equation of the axis of symmetry of f which has a negative gradient. Leave your answer in the form $y = mx + c$.

g) Sketch the graphs of f and g on the same system of axes. Clearly show ALL intercepts with the axes and any asymptotes.

h) If it is given that $f(-1) = g(-1)$, determine the values of x for which $g(x) \geq f(x)$

Teaching notes:

Ask: *What functions are represented here?*

(Hyperbola and exponential function) and what they remember about these functions (that the hyperbola has two asymptotes and that the exponential function has one asymptote).

a)

Learners should be able to substitute -3 into the function to find the corresponding value.

Say: *Explain what you have found.*

(The y-value of the function when $x = -3$).

b)

Ask: *What makes this question different from the previous question?*

(In this question, the y-value of the function has been given and the corresponding x-value is required).

c)

Ask: *In what form are the equations of the asymptotes of a hyperbola?*

(x =.. for the vertical asymptote and y =.. for the horizontal asymptote).

Ask: *Where do we read the information for the vertical asymptote?*

(The value of x that makes the denominator equal to zero).

Ask: *Where do we read the information for the horizontal asymptote?*

(The value of q – the vertical shift).

d)

Ask: *What does the range mean?*

(All the possible y-values of the function).

Remind earners that in an exponential function, the range will be linked directly to the horizontal asymptote. It will either begin with the asymptote and go on to infinity or start at negative infinity and end with asymptote.

Ask: *How do we know what type this exponential function will be?*

(It is a decreasing function but positive and therefore the asymptote will be below the function).

e)

Ask: *How do we find the intercepts of any function?*

(To find the x-intercept, make y = 0 and solve for x; To find the y-intercept, make x = 0 and solve for y).

f)

Ask: *How many axes of symmetry does a hyperbola have?* (2)

Describe the axes of symmetry.

(One axis has a positive slope with a gradient of –1; the other axis has a negative slope with a gradient of 1.

Both axes of symmetry pass through the point where the asymptotes meet).

g)

Learners should be able to do this using some of the answers from previous questions. Remind learners to draw in the asymptotes and label them and that they will need to find the intercepts of the exponential function.

h)

It is important that learners recognise the importance of their answer to a) and (b) in answering this question.

In a) learners found that $f(-3) = 4$ and in b) learners also found that $g(-3) = 4$ (even though the value of y was given and x was found).

This point of intersection is key to answering part of the question. The other point of intersection, which is also required, is given in the question.

Ask learners to highlight the part of the exponential graph that lies ABOVE (greater than) the hyperbola.

a) $f(-3) = \frac{-3}{-3+2} + 1$

$= 4$

b) $4 = 2^{-x} - 4$

$8 = 2^{-x}$

$2^3 = 2^{-x}$

$\therefore x = -3$

c) $x = -2$

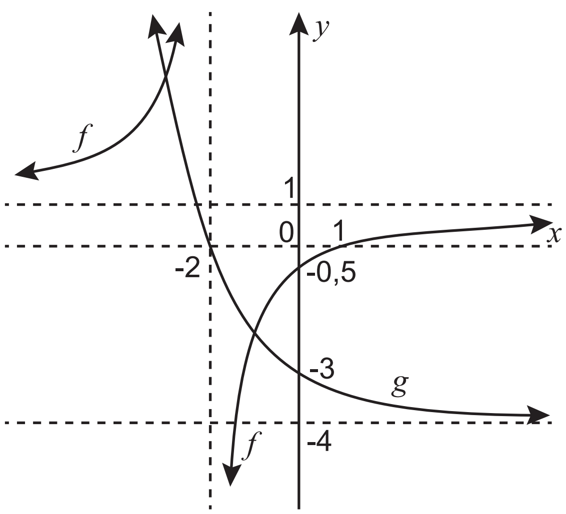
$y = 1$

d) $y > -4$

e)
$$f(x) = \frac{-3}{x+2} + 1$$
$$0 = \frac{-3}{x+2} + 1$$
$$-1 = \frac{-3}{x+2}$$
$$-1(x + 2) = -3$$
$$-x - 2 = -3$$
$$x = 1 \qquad (1;0)$$
$$y = \frac{-3}{0+2} + 1$$
$$y = -\frac{1}{2} \qquad (0;-\frac{1}{2})$$

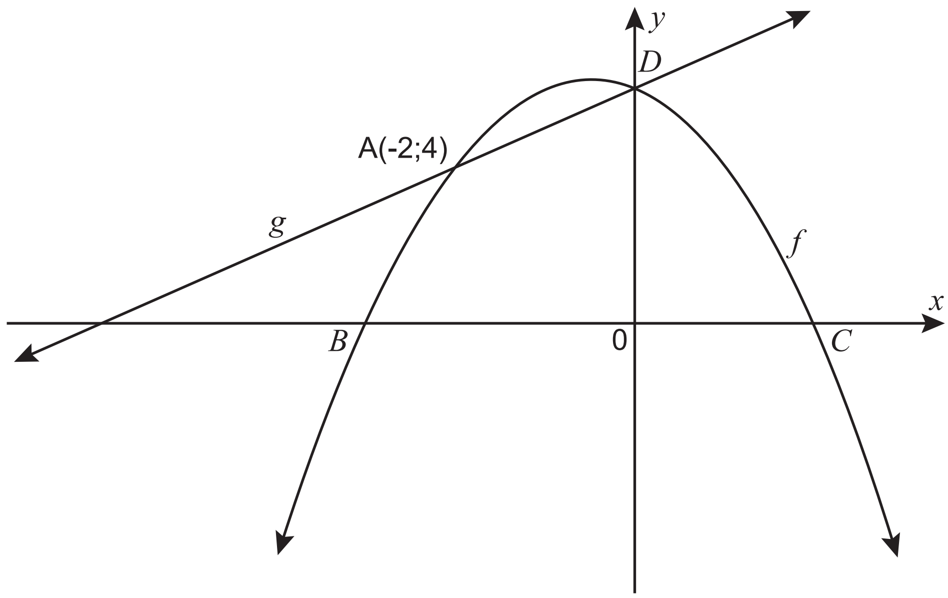
f)
$$m = -1 \qquad (-2;1)$$
$$y = -x + c$$
$$1 = -(-2) + c$$
$$-1 = c$$
$$\therefore y = -x - 1$$

g)



h) $x \le -3$ or $-2 < x \le -1$

The diagram below shows the graphs of $f(x) = -x^2 - x + 6$ and $g(x) = mx + c$.
$A(-2\ ;\ 4)$ is a point of intersection of the two graphs.



a) Determine the x-intercepts of f
b) Write down the equation of the axis of symmetry of f
c) Determine the range of f
d) Write down the equation of g in the form $g(x) = mx + c$
e) Write down the average gradient between points A and D.
f) Determine the equation of h, if h is the reflection of f about the x-axis and then translated 3 units to the right. Leave your answer in the form $h(x) = a(x + p)^2 + q$
g) Write down the values of x for which $f(x) > 0$
h) If $f(p) = f(r) = 4$, calculate the value of $p - r$ if $r < 0$.

Teaching notes:

a)

Ask: *How do we find the x-intercept(s) of any function?*

(Make $y = 0$ and solve for x)

b)

Ask: *Where is the axis of symmetry on a parabola?*

(A vertical line that passes through the turning point).

Say: *We need to find the x-coordinate of the turning point. How do we find that?*

($x = \frac{-b}{2a}$)

c)
Ask: *What does the range mean?*
(All the possible y-values of the function).
Remind learners that these values should be read from lowest to highest – so they must read from bottom to top.
Ask: *What are the first y values of the function?*
(There are arrows, so negative infinity).
Ask: *At which value does the function end (the maximum point)?*
(The y-coordinate of the turning point which we are not given. We need to calculate it using the value found in b))

d)
Ask: What is required to find the equation of a straight line?
(The gradient and a point).
Do we have this information?
(Yes – there are 2 points given – gradient can be found from these).

e)
Ask: *Why is the term 'average gradient' instead of 'gradient'?*
(The two points are on a curve and therefore cannot have an accurate gradient).
Point out that we already know the gradient between these two points from d).

f)
Ask: *What is the rule for a reflection in the x- axis?*
$(x ; y) \rightarrow (x ; -y)$
Ask: *How do we show a translation to the right?*
(Subtract the shift from the x-value. This means we need to complete the square to work inside the bracket).

g)
Remind learners to highlight the part of the function that corresponds to the inequality (in this case greater than zero – the part of the function that is positive) then find the x-values that correspond to the highlighted part of the function.

h)
Learners may find this question difficult.
Ask: What does a function look like if it is equal to a constant?
(A horizontal line).
Tell learners to draw in the line $y = 4$.
Ask: *Can you see that there are two values where the function $f(x)$ is equal to the line drawn in?*
Tell learners that we need to find these two values then use the information in the question to decide which one is p and which one is r to answer the final part of the question.

a) $0 = -x^2 - x + 6$
$0 = x^2 + x - 6$
$0 = (x + 3)(x - 2)$
$x = -3$ or $x = 2$
$(-3 ; 0)$ $(2 ; 0)$

b) $x = \frac{-b}{2a}$
$x = \frac{-(-1)}{2(-1)}$
$x = -\frac{1}{2}$

c) $f\left(-\frac{1}{2}\right) = -\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 6$
$f\left(-\frac{1}{2}\right) = 6\frac{1}{4}$
$\therefore y \leq 6\frac{1}{4}$

d) $A(-2 ; 4)$ $D(0 ; 6)$
$m = \frac{6-4}{0-(-2)}$
$m = 1$
$\therefore g(x) = x + 6$

e) Average gradient = 1

f) $f(x) = -x^2 - x+6$
$f(x) = -(x^2 + x-6)$
$f(x) = -(x^2 + x + \frac{1}{4} - 6 - \frac{1}{4})$
$f(x) = -\left[\left(x + \frac{1}{2}\right)^2 - \frac{25}{4}\right]$
$f(x) = -\left(x + \frac{1}{2}\right)^2 + \frac{25}{4}$
$h(x) = \left(x + \frac{1}{2} - 3\right)^2 - \frac{25}{4}$
$h(x) = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4}$

g) $-3 < x < 2$

h) $f(x) = 4$
$-x^2 - x + 6 = 4$
$0 = x^2 + x - 2$
$0 = (x+2)(x - 1)$
$x = -2$ or $x = 1$
$\therefore r = -2$ and $p = 1$
$\therefore p - r = 1 - (-2) = 3$

FINANCE AND GROWTH

a) A company bought machinery costing R80 000. Using the reducing balance method, the machinery had a book value of R20 000 after five years. Calculate the rate of depreciation.

b) Calculate the effective interest rate if interest is compounded at 5% p.a. compounded quarterly.

c) Sipho invested R30 000 for six years. The investment earned interest at 12% p.a., compounded monthly for the first two years. Thereafter, the interest rate changed to 10,8% p.a. compounded semi-annually for the rest of the period. Calculate the value of the investment at the end of 6 years. (No other transactions were made on the account).

d) Mary deposited R25 000 into a savings account with an interest rate of 18% p.a., compounded monthly. Mary withdrew R8 000 from the account two years after depositing the initial amount. She deposited another R4 000 into this account 3,5 years after the intial deposit. What amount will Mary have five years after making the initial deposit in this account?

Teaching notes:

a)

Learners should recognise that they need to use the reducing balance formula for depreciation.

Ask: *What are our known values?*

(A – R20 000, the current value; P – R80 000, the original value; n – 5 years).

b)

Remind learners that they need to know the formula for converting between a nominal and effective interest rate.

Ask: *What is the key point to remember when dealing with effective interest rates?*

(We deal with one year only as we are effectively finding the annual compounded rate).

c) & d)

Remind learners that timelines are a good strategy to solve more complicated questions where the interest rates change over a given time or withdrawals and extra deposits are made.

Draw the timelines with learners and remind them of the process.

a)

$$A = P(1 - i)^n$$

$$20\,000 = 80\,000(1 - i)^5$$

$$0{,}25 = (1 - i)^5$$

$$\sqrt[5]{0{,}25} = 1 - i$$

$$i = 1 - \sqrt[5]{0{,}25}$$

$$i = 0{,}24214417$$

Rate = 24,21%

b) $1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$

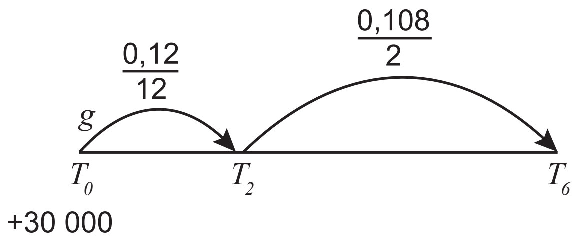
$1 + i_{eff} = \left(1 + \frac{0,05}{4}\right)^4$

$i_{eff} = \left(1 + \frac{0,05}{4}\right)^4 - 1$

$i_{eff} = 0,05094\ldots$

$\therefore$ effective rate = 5,09% p.a.

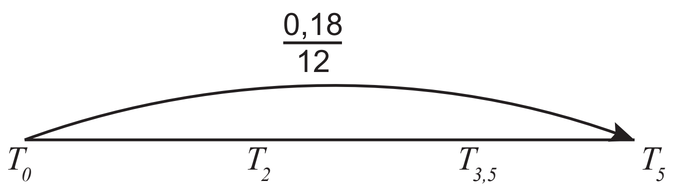
c)



$A = 30\,000\left(1 + \frac{0,12}{12}\right)^{2\times 12}\left(1 + \frac{0,108}{2}\right)^{4\times 2}$

$A = 58\,017,51$

d)



$A = 25\,000\left(1 + \frac{0,18}{12}\right)^{5\times 12} - 8\,000\left(1 + \frac{0,18}{12}\right)^{3\times 12} + 4\,000\left(1 + \frac{0,18}{12}\right)^{1,5\times 12}$

$A = 52\,636,74$

PROBABILITY

a) A bag contains three blue marbles and two red marbles. A marble is taken from the bag, the colour is recorded and the marble is put aside. A second marble is taken from the bag, the colour is recorded then put aside.

(i) Draw a tree diagram to represent the information above. Show the probabilities associated with EACH branch, as well as the posible outcomes.

(ii) Determine the probability of first taking a red marble and then taking a blue marble, in that order.

b) A and B are two events. The proability that event A will occur is 0,4 and the probability that event B will occur is 0,3. The probability that either event A or event B will occur is 0,58.

(i) Are events A and B mutually exclusive? Justify your answer with appropriate calculations.

(ii) Are events A and B independent? Justify your answer with appropriate calculations.

Teaching notes:

a)

Ensure that learners realise that the marble is drawn at random and is NOT replaced and that this will affect the probability for the second draw.

As you draw the tree diagram with learners, point out the correct method (to start with a vertex, to write the probability ON the branch and the outcome at the END of the branch).

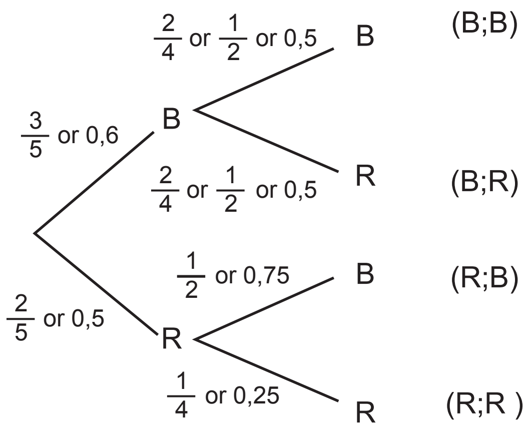
Remind learners when we multiply (along the branches to match an outcome required) and when we add the probabilities (if more than one outcome is possible – OR)

b)

Ask questions regarding the vocabulary involved in this question (mutually exclusive and independent) as well as the rules of probability – the addition rule and the rule for independent events).

Solutions:

a) (i)



(ii) $P(RB) = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$ or 0,3

b) (i) $P(A)$ = 0,4 $P(B)$ = 0,3 $P(A \text{ or } B)$ = 0,58

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$0{,}58 = 0{,}4 + 0{,}3 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0{,}12$

∴ Events A and B are not mutually exclusive as $P(A \text{ and } B) \neq 0$

(ii) $P(A \text{ and } B) = 0{,}12$

$P(A) \times P(B) = 0{,}4 \times 0{,}3$

$= 0{,}12$

∴ A and B are independent events because $P(A \text{ and } B) = P(A) \times P(B)$

A survey was done among 80 learners on their favourite sport.
The results are shown below.

- 52 learners like rugby (R)
- 42 learners like volleyball (V)
- 5 learners like chess (C) only
- 14 learners like rugby and volleyball but not chess
- 12 learners like rugby and chess but not volleyball
- 15 learners like volleyball and chess but not rugby
- x learners like all 3 types of sport
- 3 learners did not like any sport

a) Draw a Venn diagram to represent the information above.
b) Show that $x = 8$
c) How many learners like only rugby?
d) Calclate the probability that a learner, chosen randomly, likes at least TWO different types of sport?

Teaching notes:

a)

Ask: *How many events are mentioned?*
(Three – rugby, volleyball and chess)

Ask: *When drawing and completing a Venn diagram, where should we always start?*
(The intersection and work outwards).

Ask: *What value will go in the intersection for this question?*
(x)

b)

Ask: *How will we find the value of x?*
(All the values represented should total 80. Make an equation and solve).

Remind learners that if a question states 'show that', they may NOT use the solution given in the calculation.

c)

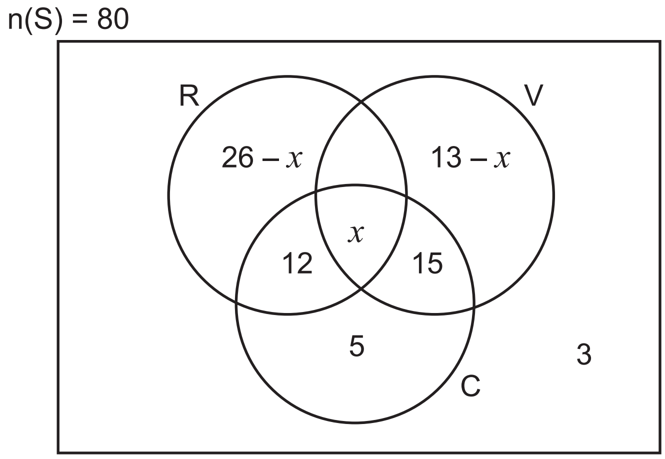
This should be a straightforward subtraction calculation using the value of x

d)

Say: *Show me on the Venn diagram where learners are represented that like at least two of the sports.*
(The areas that are shared – any intersection).

Remind learners that when answering a probability question, the sample space is essential.

a)



b) $26 - x + 14 + 13 - x + x + 12 + 15 + 5 + 3 = 80$

$x = 8$

c) Rugby only:

$26 - 8 = 18$

d) $P(\text{at least 2 types}) = \frac{12 + 14 + 15 + 8}{80}$

$= \frac{49}{80}$

8. When the past paper has been completed ask learners if they have any questions.

9. Say: *Next week we will be revising the work for Paper 2.*

TERM 4

REVISION - WEEK 2

A POLICY AND OUTCOMES

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners will have: ● worked through summaries of all Paper 2 topics ● completed a full Paper 2 with their teacher.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the summaries of Paper 2
   - work through the Paper 2 2017 examination and teaching notes.
3. The notes and examination are available in the Resource Pack for photocopying if possible. Summary: Resource 9; Paper 2 2017: Resource 10.
4. Write work on the chalkboard before the learners arrive to ensure no time is wasted.

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

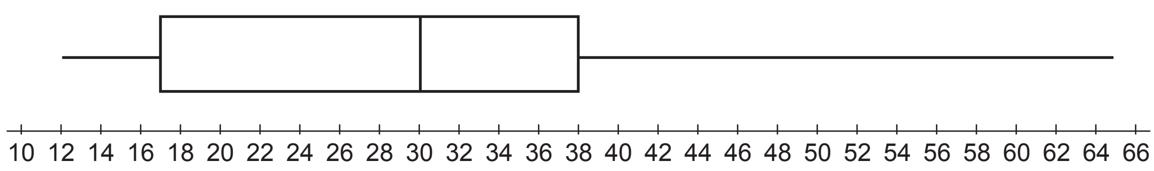
1. Support learners as they consolidate all that they have learned this year.
2. Give learners time to ask questions and become confident in their ability to write their final examination.

DIRECT INSTRUCTION

1. Hand out the four sets of summary notes for Paper 2 - Resource 9 in the Resource Pack.
2. Go through the notes with learners. This should take at least an hour.
3. Ask questions to ascertain how much learners remember as you go through each topic.
4. Encourage learners to add their own notes to the summaries – now and throughout the next few weeks of revision.
5. Once you have worked through the summary notes of all topics, hand out Paper 2 2017. Work through each question in detail with learners. Allow learners who feel confident to work on their own to do so.
6. As you go through each question, give learners the opportunity to contribute and ask questions.
7. Encourage learners to refer to their summary notes and to use them when answering questions or to add notes to if they are finding something a challenge.

STATISTICS

a) Mr Brown conducted a survey on the amount of airtime (in Rands) EACH student had on his or her cell phone. He summarised the data in the box and whisker diagram below.



(i) Write down the five-number summary of the data.
(ii) Determine the interquartile range
(iii) Comment on the skewness of the data

b) A group of 13 students indicated how long it took (in hours) before their cell phone batteries required charging. The information is given in the table below.

5	8	69	10	17	20	29	32	48	5	50	63	y	107

(i) Calculate the value of y if the mean for this data set is 41.
(ii) If $y = 94$, calculate the standard deviation of the data.
(iii) The mean time before another group of 6 students needed to recharge the batteries of their cell phones was 18 hours. Combine these groups and calculate the overall mean time needed for these two groups to recharge the batteries of their cell phones.

Teaching notes:

a)

(i) Ask: *What are the 5 items that make up the five-number summary?*
(Lowest and highest value, median, lower quartile and upper quartile).

(ii) Ask: *How do we calculate the inter quartile range?*
(Upper quartile subtract lower quartile).

(iii) Remind learners that the skewness in of a box and whisker plot can be seen by looking for the wider part of the box as this shows where the data is more spread out.

b)

(i) Ask: *How is the mean calculated?*
(By adding all the values and dividing by the number of values added).
Say: *Then we should be able to use this to find the missing value – make an equation to show that all the values are being added and divided by the number of values to equal 41*

(ii) Learners may need to be reminded of how to use their calculators to find standard deviation.

(iii) Ask: *If 6 students produced a mean average of 18 hours, what was the total of all the times?*
(6×18)
Ask: *How many students are there in total now?*
(19)

a)

(i) Minimum: 12
Q_1 : 17
M : 30
Q_3 : 38
Maximum: 65

(ii) IQR = 38 – 17 = 21

(iii) The data is negatively skewed (to the left)

b)

(i) $\frac{y + 439}{13}$ = mean

$\frac{y+439}{13} = 41$

$y + 439 = 533$

$y = 94$

(ii) $\sigma = 30{,}94$

(iii) Original sum: 41 × 13 = 533
New (group) sum: 18 × 6 = 108
Total number of students: 19
Mean time of both groups: $\frac{533 + 108}{19} = \frac{641}{19} = 33{,}74$

A student conducted a survey among his friends and relatives to determine the relationship between the age of a person and the number of marketing phone calls he or she received within one month. The information is given in the table below.

Age of person in survey	Frequency	Cumulative frequency
$20 < x \leq 30$	7	7
$30 < x \leq 40$		27
$40 < x \leq 50$	25	
$50 < x \leq 60$		64
$60 < x \leq 70$		72
$70 < x \leq 80$	4	
$80 < x \leq 90$		80

a) Complete the frequency and cumulative frequency columns in the table.
b) How many people participated in this survey?
c) Write down the modal class.
d) Draw an ogive (cumulative frequency graph) to represent the data.
e) Determine the percentage of marketing calls received by people older than 54 years.

Teaching notes:

a)

Ask learners to assist you in completing the table – ask a question before filling in each value. Learners may need to be reminded that the final column is the accumulated total.

b)

Remind learners that the final amount in the cumulated frequency column represents the total.

c)

Ask: *What is the mode?*

(The value that appears the most often).

Ask: *Why are we being asked for the modal class?*

(We don't know the actual values – only how many values are in each class interval).

d)

Remind learners of the important points regarding the drawing of an ogive:

- The horizontal axis will represent the data. In this case, age.
- Only the upper boundary numbers will be represented – show them all. These are the x-co-ordinates of the points found.
- The vertical axis will always represent the cumulative frequency – no matter what situation is represented.
- To choose a reasonable scale, take the highest number and divide by 10 – this gives an idea what multiples to use. In this case, $80 \div 10 = 8$. 10 can be used.
- Plot the points and join them freehand and as smoothly as possible – remember it is called a cumulative frequency curve.
- Remember to ground the ogive using the lower boundary number of the first class interval with zero. In this case (20 ; 0).

e)

Ask: *Where do we look on the ogive to read off this answer?*

(Find 54 on the axes and read off the corresponding y-value that represents the number of people).

Point out that the question says 'older than' so we are interested in how many people are past that value. This requires a subtraction calculation using the total number of people.

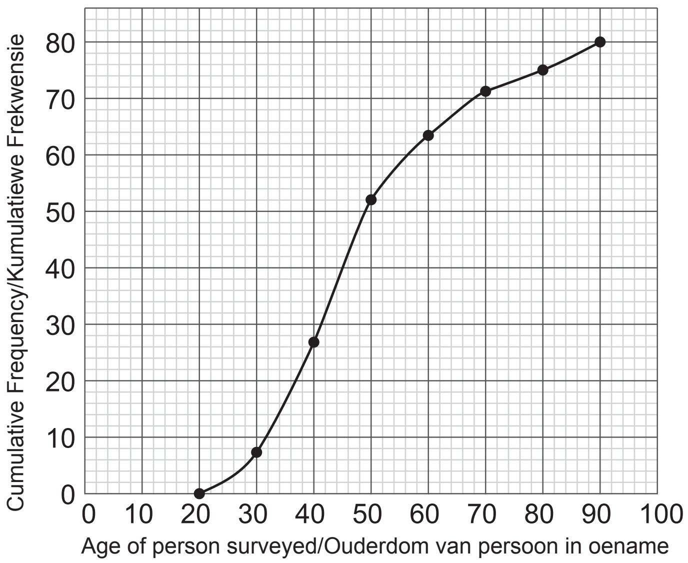
a)

Age of person in survey	**Frequency**	**Cumulative frequency**
$20 < x \leq 30$	7	7
$30 < x \leq 40$	**20**	27
$40 < x \leq 50$	25	**52**
$50 < x \leq 60$	**12**	64
$60 < x \leq 70$	**8**	72
$70 < x \leq 80$	4	**76**
$80 < x \leq 90$	**4**	80

b) 80

c) $40 < x \leq 50$

d)



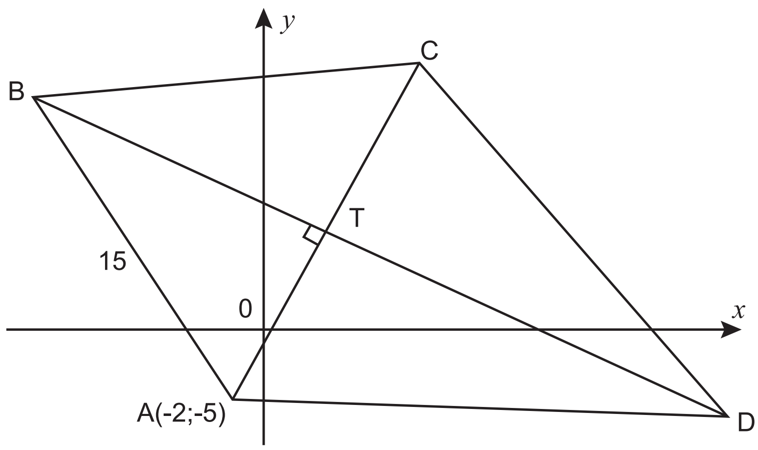
e) 80 - 58 = 22

$\frac{22}{80} = 27,5\%$

ANALYTICAL GEOMETRY

$A(-2;-5)$, B, C and D are the vertices of quadrilateral ABCD such that diagonal AC is perpendicular to diagonal BD at T.

The equation of BTD is given by $2y + x = 18$ and $AB = 15$ units.



a) Determine the gradient of line AC.
b) Determine the equation of AC in the form $y = mx + c$
c) If the equation of AC is $y = 2x - 1$, calculate the coordinates of T.
d) If ABCD is a kite with $AB = BC$:
   (i) Determine the coordinates of C
   (ii) Calculate the length of BT
   (iii) Write down the length of the radius of the circle passing through points B, C and T

Teaching notes:

a)

Ask: *How will we find the gradient of AC?*

(The equation of BD is given, and we therefore know the gradient. AC is perpendicular to BD – this will be useful to find the gradient).

b)

Ask: *What do we need to find the equation of any line?*

(The gradient and one point).

Ask: *Do we have this?*

(Yes – we found the gradient in the previous question and point A is given).

c)

Ask: *What happens at T?*

(T is the point of intersection of the two straight lines).

How will we find T?

(Make the equations equal and solve for x and y).

d) (i) Ask: *Why is it important that ABCD is a kite?*

(Diagonal BD bisects diagonal AC which means that T is the midpoint of AC).

Ask: *How can we use this to find C?*

(Use the midpoint formula in reverse to find x and y).

(ii) Ask: *How can we find the length of BT?*

(Use the distance formula to find AT then the theorem of Pythagoras to find BT).

(iii) Point out that it is important to have a knowledge of all topics in mathematics and not to focus on one topic at a time. Euclidean geometry will be useful for this question.

Ask: *Which circle theorem may be useful to answer the question?*

(BC subtends a right angle and therefore must be a diameter).

a) BD: $2y + x = 18$

$$2y = -x + 18$$

$$y = -\frac{1}{2}x + 9$$

$$m_{BD} = -\frac{1}{2}$$

$$\therefore m_{AC} = 2$$

b) $A(-2; -5)$ $m = 2$

$$y - y_1 = m(x - x_1)$$

$$y - (-5) = 2(x - (-2))$$

$$y + 5 = 2(x + 2)$$

$$y = 2x + 4 - 5$$

$$y = 2x - 1$$

c) $y = -\frac{1}{2}x + 9$ and $y = 2x - 1$

$$-\frac{1}{2}x + 9 = 2x - 1$$
$$-x + 18 = 4x - 2$$
$$-x - 4x = -2 - 18$$
$$-5x = -20$$
$$x = 4$$
$$y = 2(4) - 1$$
$$y = 7 \qquad \therefore T(4;7)$$

d)

(i) $\frac{-2 + x}{2} = 4$

$$-2 + x = 8$$
$$x = 10$$
$$\frac{-5 + y}{2} = 7$$
$$-5 + y = 14$$
$$y = 19$$
$$\therefore C(10;19)$$

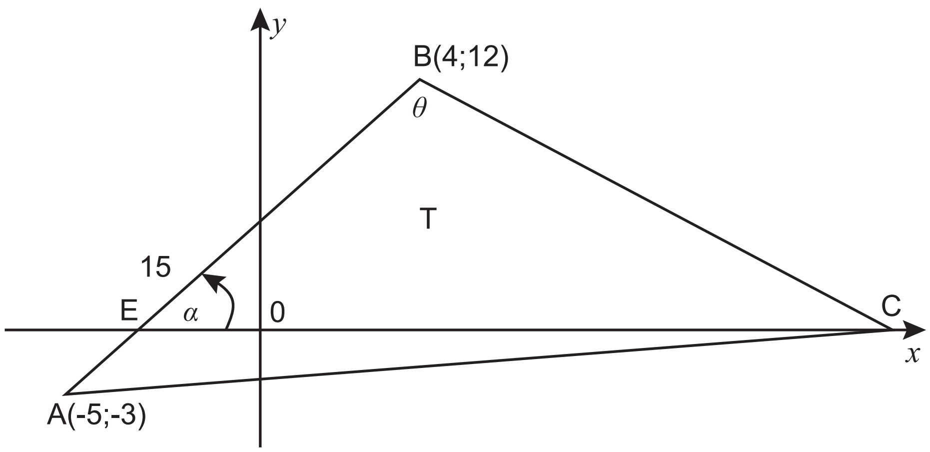
(ii) $AT = \sqrt{(4 - (-2))^2 + (7 - (-5))^2}$

$$AT = \sqrt{180} = 6\sqrt{5}$$
$$BT^2 + AT^2 = AB^2 \qquad \text{(Pythagoras)}$$
$$BT^2 + (6\sqrt{5})^2 = (15)^2$$
$$BT^2 = 225 - 180$$
$$BT = 3\sqrt{5}$$

(iii) $B\hat{T}C = 90°$ $\therefore BC$ is the diameter (conv < in semi-circle)

$BC = 15$ $\therefore rad = 7{,}5$

C, a point on the x-axis, A(-5;-3) and B(4;12) are the vertices of a triangle.
AB intersects the x-axis at E.
$A\hat{B}C = \theta$ and $B\hat{E}C = \alpha$



a) Calculate the gradient of AB.
b) Determine the coordinates of point E.
c) Determine the size of α. Round off to the nearest whole number.
d) If θ = 76º, determine the equation of the line through A parallel to BC.

Teaching notes:

a)

This should be a straightforward question as two points are given.

b)

Ask: *What is the significance of point E?*

(It is the x-intercept of the line AB).

Say: *That means we need to find the equation of the line in order to find the x-intercept.*

c)

Ask: *How do we find the angle of inclination?*

(tan $\theta = m$).

d)

Point out that to find a line parallel to BC we need to find the gradient of BC.

Ask: *How can we find the gradient of BC?*

(By finding $B\hat{C}X$ and using it to find the gradient).

Once the gradient has been found, the equation of the line can be found.

a) $m = \dfrac{12-(-3)}{4-(-5)}$

$m = \dfrac{15}{9} = \dfrac{5}{3}$

b) (4;12) $m = \dfrac{5}{3}$

$y - 12 = \dfrac{5}{3}(x - 4)$

$0 = \dfrac{5}{3}x - \dfrac{20}{3} + 12$

$0 = 5x - 20 + 36$

$-16 = 5x$

$x = \dfrac{-16}{5}$ $E\left(\dfrac{-16}{5};0\right)$

c) $\tan\alpha = \dfrac{5}{3}$

$\therefore \alpha = 59^\circ$

d) $B\hat{C}X = 135^\circ$ (ext < ΔEBC)

$\tan 135^\circ = -1$

$\therefore m_{BC} = -1$

$A(-5;-3)$ $m = -1$

$y - (-3) = -1(x - (-5))$

$y + 3 = -x - 5$

$y = -x - 8$

TRIGONOMETRY

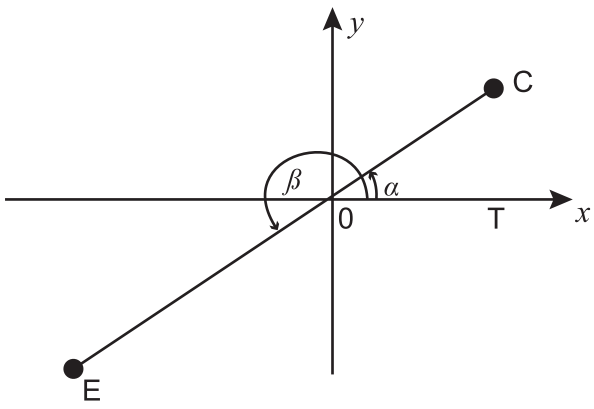
a) Simplify fully: $\sin(90° - x).\cos(180° + x) + \tan x.\cos x.\sin(x - 180°)$

b) Prove, WITHOUT using a calculator, that

$$\frac{\sin 315°.\tan 210°.\sin 190°}{\cos 100°.\sin 120°} = \frac{-\sqrt{2}}{3}$$

c) In the diagram below, R is a point in the first quadrant such that $T\hat{O}R = \alpha$. RO is extended to P such that $OP = 2RO$ and $T\hat{O}P = \beta$.

It is given that $\sin \alpha = \frac{3}{5}$



WITHOUT the use of a calculator, determine:

(i) The value of $\tan \alpha$

(ii) The value of $\sin \beta$

(iii) The coordinates of P

d) Prove the identity

$$\frac{\sin\theta - \tan\theta.\cos^2\theta}{\cos\theta - 1 + \sin^2\theta} = \tan\theta$$

Teaching notes:

a)

Remind learners that they need to be proficient in reductions as well as have an understanding of complementary angles (co-ratios). It is important that they are able to find in which quadrant an angle lies and then to know whether the trigonometric function is positive or negative in that quadrant.

b)

Say: *If reductions of angles are required without a calculator, remember to expect special angles.*

c)

(i) Ask: *What type of question is this?*

(Pythagoras).

(ii) Say: *Describe the relationship between angles β and α.*

((β = 180º + α). This can be used to find sin β).

(iii) Ask: If RO = 5, what is the length of OP? (10 units)

Point out that because vertically opposite angles are equal, we have two similar triangles.

Ask: *What do you know about similar triangles?*

(Sides are in proportion).

Say: *If the sides are in proportion, this can be used to find y.*

d)

Remind learners of the tips to prove identities:

- Change tan θ to $\frac{sin\theta}{cos\theta}$
- Simplify where possible
- Look for factorising opportunities.

a) $sin\,(90º - x).cos\,(180º + x) + \tan x.cos\,x.sin\,(x - 180º)$

$= \cos x.(-\cos x) + \frac{\sin x}{\cos x}.\cos x.(-\sin x)$

$= -\cos^2 x - \sin^2 x$

$= -(\cos^2 x + \sin^2 x)$

$= -1$

b) $\text{LHS} = \frac{(sin315°.tan210°.sin190°)}{cos100°.sin120°}$

$= \frac{sin(360° - 45°).tan(180° + 30°).sin(180° + 10°)}{cos(180° - 80°).sin(180° - 60°)}$

$= \frac{-sin45°.tan30°.-sin10°}{-cos80°.sin60°}$

$= \frac{-\frac{\sqrt{2}}{2}.\frac{1}{\sqrt{3}}.-sin10°}{-cos\,80°.\frac{\sqrt{3}}{2}}$

$= \frac{\frac{\sqrt{2}}{2\sqrt{3}}}{-\frac{\sqrt{3}}{2}}$

$= -\frac{\sqrt{2}}{3}$ = RHS

c) (i) $OT^2 + RT^2 = OR^2$ (Pythagoras)

$x^2 + 3^2 = 5^2$

$x = 4$

$\therefore \tan\alpha = \frac{3}{4}$

(ii) $\sin\beta = \sin(180° + \alpha)$

$= -\sin\alpha$

$= -\frac{3}{5}$

(iii) $\frac{y}{10k} = \frac{-3k}{5k}$

$5y = -30k$

$y = -6k$

$\therefore x = -8k$ $\therefore P(-8k;-6k)$

d) LHS = $\frac{sin\theta - tan\theta.cos^2\theta}{cos\theta - 1 + sin^2\theta}$

$= \frac{sin\theta - \frac{sin\theta}{cos\theta}.cos^2\theta}{cos\theta - (sin^2\theta + cos^2\theta) + sin^2\theta}$

$= \frac{sin\theta - sin\theta.cos\theta}{cos\theta - sin^2\theta - cos^2\theta + sin^2\theta}$

$= \frac{sin\theta(1 - cos\theta)}{cos\theta - cos^2\theta}$

$= \frac{sin\theta(1 - cos\theta)}{cos\theta(1 - cos\theta)}$

$= \frac{sin\theta}{cos\theta}$

$= \tan\theta = RHS$

a) Determine the general solution for $\sin(x - 30°) = \cos 2x$

b) Consider the functions $f(x) = \sin(x - 30°)$ and $g(x) = \cos 2x$

(i) Write down the period of g

(ii) State the range of f

(iii) Draw the graphs for f and g for $x \in [-90°; 180°]$. Clearly show all intercepts with the axes, turning points and end points.

(iv) Write down the x-coordinates of the points of intersection of f and g in the interval $x \in [-90°; 180°]$.

Teaching notes:

a)

Ask: *How do we deal with equations with sine and cosine functions of different angles?*

(Use co-functions to get the same trig function on each side of the equation).

b)

(i) Ask: *What is meant by the term period?*

(The distance required for the function to complete a full cycle).

Ask: *What is the period of g(x)?*

(180º)

(ii) Ask: *What is meant by the term range?*

(The set of all output values of a function).

(iii) Remind learners that they need to know their basic graphs well and have a good understanding of the transformations possible.

(iv) The general solution found in a) will be used to find the points of intersection of the functions.

a) $\sin(x - 30°) = \cos 2x$

$\sin(x - 30°) = \sin(90° - 2x)$

$\therefore x - 30° = 90° - 2x + k.360°$ or $x - 30° = 180° - (90° - 2x) + k.360°$

$3x = 120° + k.360°$ $\quad$ $-x = 120° + k.360°$

$x = 40° + k.120°$ $\quad$ $x = -120° - k.360°$

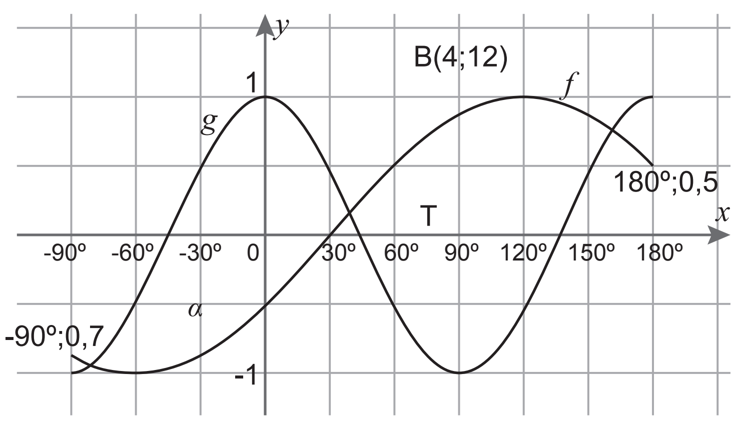
$k \in Z$

b)

(i) 180º

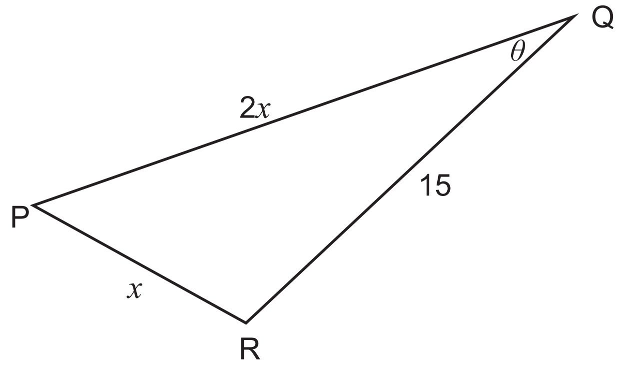
(ii) $-1 \leq y \leq 1$

(iii)



(iv) $x = -80°$; $x = 40°$; $x = 160°$

In $\triangle PQR$, QR = 3 units, $PR = x$ units, $PQ = 2x$ units and $P\hat{Q}R = \theta$



a) Show that $\cos\theta = \frac{x^2+3}{4x}$

b) If x = 2,4 units:

(i) Calculate θ

(ii) Calculate the area of $\triangle PQR$

c) Calculate the values of x for which the triangle exists

Teaching notes:

a)

Ask: *Which rule is used when three sides of a triangle are given?*

(The cosine rule).

b)

(i) Say: *This is a substitution question using the information from a) which leads to solving an equation.*

(ii) Using the size of the angle found, this will be a straightforward substitution into the area rule formula.

c)

Tell learners to consider the lengths of the sides. QR = 3; PR = 2,4; PQ = 4,8

Say: *Now we need to consider the relationship between the sides in terms of x (which sides are bigger or smaller) than others and create inequalities to solve for x.*

a)

$$x^2 = (2x)^2 + (3)^2 - 2(2x)(3)\cos\theta$$

$$x^2 = 4x^2 + 9 - 12x\cos\theta$$

$$-3x^2 - 9 = -12x\cos\theta$$

$$3x^2 + 9 = 12x\cos\theta$$

$$\frac{3x^2+9}{12x} = \cos\theta$$

$$\frac{3(x^2+3)}{12x} = \cos\theta$$

b)

(i) $\frac{x^2+3}{4x} = \cos\theta$

$\frac{(2,4)^2+3}{4(2,4)} = \cos\theta$

$\cos\theta = 0,9125$

$\therefore \theta = 24,15°$

(ii) Area $\Delta PQR = \frac{1}{2}(PQ)(QR)\sin\hat{Q}$

$= \frac{1}{2}(4,8)(3)\sin 24,15°$

$= 2,95\ units^2$

c) $x + 2x > 3$ and $x + 3 > 2x$

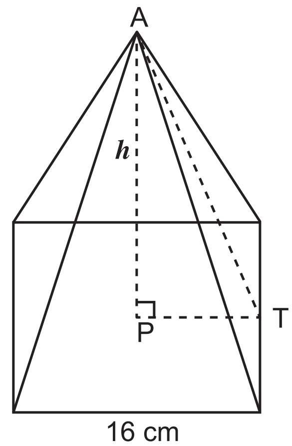
$3x > 3$ $\quad -x > -3$

$x > 1$ $\quad x < 3$

$\therefore 1 < x < 3$

MEASUREMENT and EUCLIDEAN GEOMETRY

A pyramid with a square base with a side length of $16cm$ is sketched below.
P lies on the square base directly below A. The volume of the pyramid is $640cm^3$.
Volume of a pyramid = $\frac{1}{3}Ah$.



a) Show that the perpendicular height of the pyramid, AP, is $7,5cm$.

b) Hence, determine the total surface area of the pyramid.

Teaching notes:

Ask: *What does 'directly below' tell you?*

(It is the perpendicular height).

a)

Remind learners that if a question's instruction is to 'show that' (or prove that) and the answer is given in the question, they must be very careful how they answer. They may NOT use the answer given in their solution.

Tell learners to rather imagine that the question has asked: 'find the height' and to use the fact that the height is given as an opportunity to check their answer.

Ask: *How will you 'find' the height?*

(Volume is given so use the volume formula – which is given – and fill in all known details then solve for height).

b)

Discuss the word 'hence' with learners. Tell learners that it means they should use the information from the previous question to solve this question. Point out that learners could do this question with the information given in the previous once even if they did not manage to get that right.

Ask: *How will you find surface area of the pyramid?*

(Find the area of the square base and the area of the four triangles and add them together).

Ask: *How will you find the height of the triangle?*

(The height of the triangle is the slant height of the pyramid which can be found using the theorem of Pythagoras).

Solution:

a)

$$\text{Volume} = \frac{1}{3} Ah$$

$$640cm^3 = \frac{1}{3}(16cm)(16cm)h$$

$$\frac{640cm^3}{\frac{1}{3}(16cm)(16cm)} = h$$

$$\therefore h = 7{,}5cm$$

b) Slant height:

$$s^2 = 8^2 + (7{,}5)^2$$

$$s^2 = 120{,}25$$

$$\therefore s = 10{,}9658\ldots$$

Surface area = area of base + 4 triangles

$$= l^2 + 4\left(\frac{1}{2}bh\right)$$

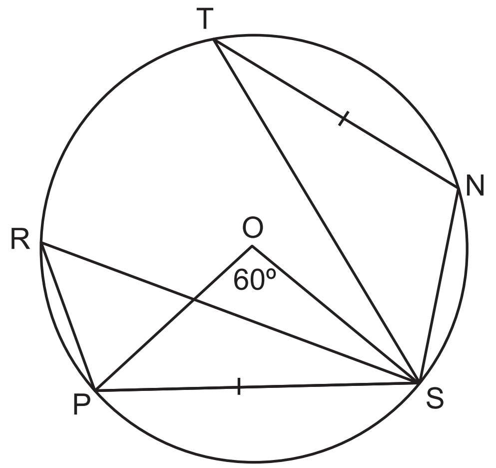
$$= (16)^2 + 2(16)(10{,}9658\ldots)$$

$$= 606{,}91cm^2$$

NOTE: Because the Euclidean geometry questions from the 2017 final examination were used in the revision lesson at the end of the lesson plans, the following questions are taken from the 2016 final examination.

a) Complete the statement so that it is TRUE: The angle subtended by an arc at the centre of the circle is …

b) O is the centre of the circle TNSPR. $P\hat{O}S = 60^\circ$ and $PS = NT$.



Calculate the size of:

(i) $P\hat{R}S$

(ii) $N\hat{S}T$

Teaching notes:

a)

Point out that knowing their theory is always important and that whatever theorem is being assessed in the theory will be required in the following part of the question.

b)

Ask: *What is the size of* $P\hat{R}S$*?* (30º). *Why?*

Ask: *What connection does the unknown angle have to any of the other angles?*

($N\hat{S}T$ is subtended from a chord that is equal in length to the chord that subtends $P\hat{R}S$

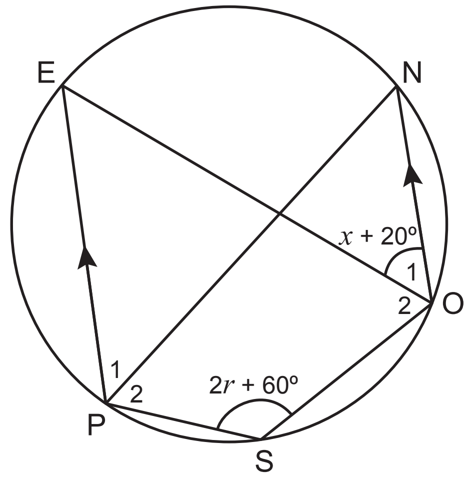
a) …equal to twice the angle subtended by the arc at the circumference.

b)

(i) $P\hat{R}S = 30^\circ$ (< at centre = 2x < at circumference)

(ii) $N\hat{S}T = 30^\circ$ (equal chords subtend equal angles)

D, E, F, G and H are points on the circumference of a circle. $\hat{G}_1 = x + 20°$ and $\hat{H} = 2x + 10°$. DE || FG.



a) Determine the size of $D\hat{E}G$ in terms of x.
b) Calculate the size of $D\hat{H}G$

Teaching notes:
Ask learners if there is any information in the question that needs to be transferred onto the diagram or anything they can fill in that they already know.
Point out the parallel lines mentioned. Remind learners that information is not given unless it is useful. Learners will therefore be expected to use at least one of the parallel line theorems that they learned in Grade 8. Ask learners to remind you what the parallel line theorems are.
Say: *Name any cyclic quads you can see.*
($DFGH$ and $EDHG$).

Solution:

a) $D\hat{E}G = x + 20°$ (alt <'s equal; $DE \parallel FG$)

b) $\hat{F} = x + 20°$ (<'s in same segment)

$2x + 10° + x + 20° = 180°$ (opp <'s of cyclic quad)

$3x + 30° = 180°$

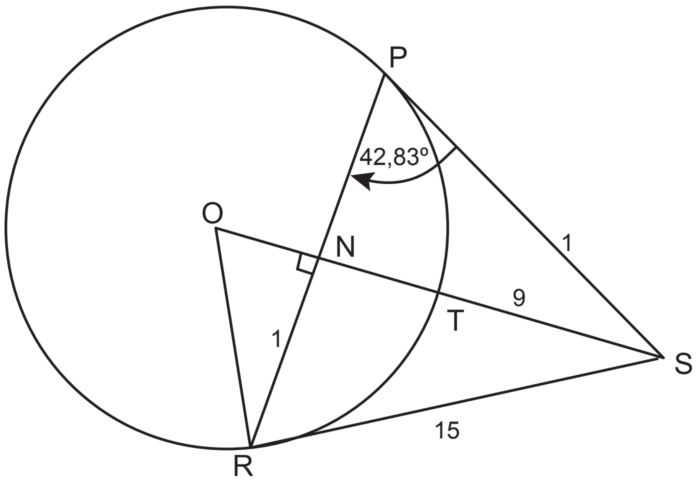
$3x = 150°$

$x = 50°$

$\therefore D\hat{H}G = 2(50°) + 10°$

$\therefore D\hat{H}G = 110°$

O is the centre of the circle PTR. N is a point on chord RP such that $ON \perp PR$.
RS and PS are tangents to the circle at R and P respectively.
RS = 15 units; TS = 9 units; $R\hat{P}S$ = 42,83º



a) Calculate the size of $N\hat{O}R$
b) Calculate the length of the radius of the circle.

Teaching notes:
Remind learners that any information given in a geometry rider will be useful to answer any question.
Ask: *What theorems do you recognise from the information or the diagram?*
(Equal tangents from common point – which leads to an isosceles triangle; tangent perpendicular to radius).
Say: *Describe a strategy to answer the questions.*
a)
(Isosceles triangle, equal angles, tangent perpendicular to radius, angles of a triangle)
b)
Use a variable for the radius and values of other sides to use Pythagoras and solve for x.

a) $SP = SR$ (tans from common point)
$\therefore S\hat{R}P = S\hat{P}R$ = 42,83º (<'s opp equal sides)
$O\hat{R}S$ = 90º (tan⊥rad)
$\therefore O\hat{R}N$ = 47,17º
$\therefore N\hat{O}R$ = 42,83º (int <'s Δ)

b) Let OR = OT = x

$\therefore$ OS = $x + 9$

$x^2 + 15^2 = (x + 9)^2$ (Pythagoras)

$x^2 + 225 = x^2 + 18x + 81$

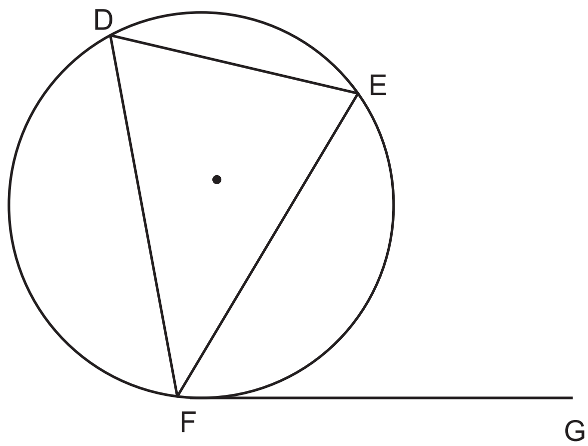
$225 - 81 = 18x$

$144 = 18x$

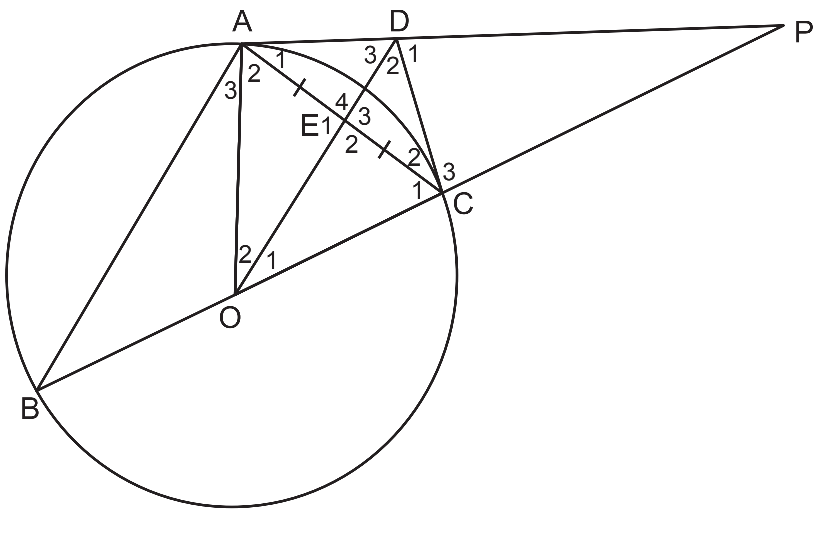
$8 = x$

$\therefore$ The radius is 8 units

a) Use the diagram to prove the theorem which states that $E\hat{F}G = E\hat{D}F$.



b) In the diagram below, BOC is a diameter of the circle. AP is a tangent to the circle at A and $AE = EC$.



Prove that:

(i) BA || OD

(ii) AOCD is a cyclic quadrilateral

(iii) DC is a tangent to the circle at C.

Teaching notes:

a)

Discuss again the importance of knowing their theory and that if theory is asked, this theorem will be required to answer any further questions.

b)

Remind learners that when required to prove something, this may NOT be used in the solution.

(i) Ask: *If lines ARE parallel, which angles have a relationship?*

(Alternate angles are equal; corresponding angles are equal and co-interior angles are supplementary).

Say: *Then this is what we need to find to prove that the lines are parallel but using OTHER theorems.*

(ii) Ask: *What are the three ways that we can prove that a quadrilateral is cyclic?*

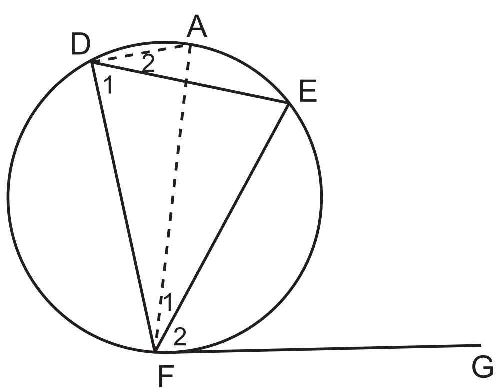
(One pair of opposite angles are supplementary; exterior angle is equal to opposite interior angle; equal angles from same line segment – the converse of equal angles in the same segment).

Ask learners to look at the diagram now and find one of these possibilities.

(iii) Ask: *What needs to be true for DC to be a tangent to the circle?* ($\hat{C}_2$ would need to be equal to $\hat{B}$ OR DC would need to be perpendicular to OC).

Ask learners to look at the diagram now and find one of these possibilities.

a) Construct diameter FOA and join AE.



$A\hat{F}E = \hat{F}_1$ and $E\hat{F}G = \hat{F}_2$; $F\hat{D}E = \hat{D}_1$ and $A\hat{D}E = \hat{D}_2$

$\hat{D}_1 + \hat{D}_2 = 90°$ (< in semi-circle)

$\hat{F}_1 + \hat{F}_2 = 90°$ (rad $\perp$ tan)

$\therefore \hat{D}_1 + \hat{D}_2 = \hat{F}_1 + \hat{F}_2$

but $\hat{F}_1 = \hat{D}_2$ (<'s in same segment)

$\therefore \hat{F}_2 = \hat{D}_1$

$\therefore E\hat{F}G = EDF$

b)

(i)	$B\hat{A}C = 90º$	(< in semi-circle)
	$\hat{E}_1 = 90º$	(line from centre to midpoint chord)
	$\therefore BA \parallel OD$	(co-int <’s supplementary)
(ii)	$\hat{A}_1 = \hat{B}$	(tan chord theorem)
	$\hat{O}_1 = \hat{B}$	(BA || OD; corres <’s equal)
	$\therefore \hat{A}_1 = \hat{O}_1$	
	$\therefore AOCD$ is a cyclic quadrilateral	(converse <’s in same segment)
(ii)	$O\hat{C}D = 90º$	(opp <’s cyclic quad)
	$\therefore CD$ is a tangent	(converse tan$\perp$rad)

8. When the past paper has been completed ask learners if they have any questions.

9. Say: *Next week you will be working through your own past papers.*

TERM 4

REVISION - WEEK 3

POLICY AND OUTCOMES

A

CAPS Page Number	39
Lesson Objectives By the end of the lesson, learners will have: ● completed a Paper 1 past paper ● completed a Paper 2 past paper ● made ‘cheat sheets’ covering all topics.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the past examination papers that the learners will be doing on their own. This is essential to assist them quickly and smoothly when learners need help.
   - work through the instructions to learners on how to make a cheat sheet to assist them in their studying.
3. The examinations are both available in the Resource Pack (RESOURCES 11-14) for photocopying if possible.

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners have now done revision with your help. It is time for them to try past papers on their own.
2. As learners work through the past papers they should make ‘cheat sheets’. Explain how to go about this before they start on the past papers.

DIRECT INSTRUCTION

1. Start the lesson by saying: Now that we have spent two weeks doing revision together, it is time for you to work on your own. I will be available to assist you but mostly you need to work alone.
2. Tell learners that while they are working through the papers, they should make cheat sheets. Make it clear that cheat sheets are a study aid and not notes that can be used for cheating!
3. Go through the following instructions with learners.

A cheat sheet is a document (generally only one-page front and back) that contains all the key information that is likely to be in an assessment. Even though you can't use the cheat sheet in the examination, the preparation of a cheat sheet is a great way to prepare for the exam.

Guidelines for preparing a cheat sheet:

1. Develop the cheat sheet gradually by adding new items as you work through past papers.
2. Write out the cheat sheet by hand. You can get more on the document that way. At the end of each past paper, re-do the cheat sheets for each topic and put them in a safe place with your summary notes as well as any other study notes. .
3. Include the following items on your cheat sheet.
   - Formulas
   - Example problems worked out
   - Steps used in the problem listed in order
   - Reminders of things to look out for in doing a problem
   - Rules used to solve problems
   - Definitions
   - Types of problems that you know will be in an examination.
4. If you recall problems you struggled with in the past, be sure to include information on these.
5. Use your past papers as a guideline to prepare your cheat sheets. Past papers are all set along the same lines. This is your 2nd past paper for each exam (Paper 1 and Paper 2), so you should start noticing what is often assessed.
6. While you are working through the past paper, refer to your exercise book, text book and summary notes for cheat sheet information.
7. Find a method to compartmentalise items. For example, highlight what you need to memorise in one colour and tips in another colour. Use bullet points and different-sized headings. Find a layout that suits your study method.
8. Choose whether your cheat sheet is a summarised list (like the summary notes you have already received) or a mind map.

9. Review your cheat sheet and summary notes for at least one hour every day for a week before the exam. This continual review will help you remember the concepts.
10. Use your cheat sheet as the primary study resource for the final. If you have kept these up-to-date, you should be able to reduce your preparation time for finals.

4. Hand out both the past papers (Paper 1 and Paper 2 – Resources 11 and 13).
   Allow learners to choose where they start. Point out, however, that by the end of the seven or eight days they must have completed both papers as well as their cheat sheets. You will need a few days for the past papers to be marked and corrected.

5. If photocopying is readily available, you can photocopy a few memoranda (Resources 12 and 14). With two or three days to go allow learners to sit in groups to mark and discuss the solutions. If photocopying is not an option, the learners could be given the link (providing some of them have data) and marking could still take place in groups.

6. If it is not possible to photocopy or to use the electronic version, you can call out the final answers of each question then ask learners which they would like you to do in full with them.

7. Verbalising problems and sharing their own knowledge with others can be a very effective learning tool.

8. The correcting of the papers is an essential part of the exercise. There is little value in doing questions and not knowing whether they have been done correctly. When marking, tell learners that they must do the full corrections for any question that was incorrect.

9. Learners should add notes to their cheat sheets relating to any mistakes they made.

10. We wish you and your learners well.